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For the Project

INTEGRATED DOD VOICE & DATA NETWORKS AND GROUND PACKET RADIO TECHNOLOGY

VOLUME 4

GROUND PACKET RADIO TECHNOLOGY

network analysis corporation

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VOLUME 4

GROUND PACKET RADIO TECHNOLOGY

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MARKOV CHAIN INITIALIZATION MODELS FOR PACKET RADIO NETWORKS

8.1 INTRODUCTION

The problem of initialization of repeaters in packet radio networks is addressed in this Chapter and in Chapter 9. Repeaters in the radio network need to be initialized by the station; namely, to receive a so-called "label" which enables addressing specific devices during packet transportation. It is assumed that a priori the station is not informed of the existance and location of specific repeaters. Hence, the initialization process involves sending special control packets, called Repeater On Packets (ROP's) from repeaters to the station to inform existance and location, the transmission of labels by the station to repeaters identified, and the transmission of End-to-End Acknowledgments (ETE Ack's) by repeaters which receive a label. A repeater is considered by the station to be initialized after the station receives an ETE Ack to the label packet.

Approximate models for initialization were developed and studied in [NAC, 1976]. These models addressed a single hop packet radio network with general interference patterns of repeaters. The initialization time was studied in [NAC, 1976] as a function of station and repeater transmission rates. However, the above models were approximate, in that the exact state of initialization of the system or station buffer size and management were not taken into account, neither was an exact initialization scenario employed; since the objective was to study general trends of initialization time as a function of network size and operating parameters. Hence, conservative, worst case *initialization times were obtained.*

In this Chapter we discuss refined models that provide a more accurate description of the initialization process. Also, the models allow us to analyze the case of a finite buffer in the station with essentially no additional complexity. The computational complexity of the models in this Chapter are significantly greater than

for the models given in [NAC, 1976], thus only small networks (number of repeaters $\leq$ 7) with complete interference have been studied.

In Section 3 we develop a Markov Chain initialization model for a single hop complete interference system (i.e., all devices are within an effective transmission range of each other, hence the simultaneous transmission by more than one device will result in an interference). The computational complexity of the model is obtained and numerical studies of initialization time performed. In Section 4 we present a Markov Chain model for a general interference system and derive its complexity. The model appears to be too complex for numerical investigations. The initialization problem is stated in Section 2.

The models in this Chapter assume that when the station has more than one label (for transmission) in its queue, the label to be transmitted is selected at random, each time. The models in Chapter 9 assume a First-In-First-Out (FIFO) label queue management discipline; this results in a system with reduced computational complexity and enables to address larger size systems (number of repeaters).

8.2 PROBLEM DEFINITION

The packet radio system, related properties and the initialization scenario are presented by the outline below:

1. Single station, S, and m repeaters, R_j, j=1,...,m.

2. Each repeater can communicate with the station (1 hop system).

3. Each repeater can communicate with all other repeaters (complete interference). This requirement is modified in Section 4.

4. Zero propagation time of signals between devices.

5. Single channel slotted Aloha access scheme. That is, all repeaters transmit and receive (hence the station transmits) on the same channel, consequently interference between any two devices is possible. Furthermore, the slotted Aloha channel access scheme is used [ABRAMSON, 1973].

6. Zero capture receivers. That is, if more than one packet is transmitted into a slot, it is assumed that all packets will be received in error.

7. When forwarding ROP's, each repeater broadcasts into a given slot with probability p.

8. No repeater accepts repeater on packets (ROP's) or end-to-end acknowledgments (ETE Ack's) from other repeaters.

9. If a repeater receives its label from the station, it forwards one ETE Ack into the next slot and halts transmission of ROP's.

10. After the station receives a ROP from a particular repeater for the first time, the station prepares a label for that repeater and places it in the label queue. We assume this requires zero processing time.

11. The station maintains the repeater label in the queue until it forwards the label to the repeater and receives an ETE Ack, indicating the label has been received. When this occurs, the repeater is considered labeled.

12. When the label queue is not empty, the station transmits a label with probability q.

13. When transmitting a label, the station randomly selects a label from the queue. We assume it is equiprobable that any label is chosen, thus the probability of choosing any particular label is

$$\frac{1}{\text{(number of labels in the queue)}}.$$

14. Whenever the station receives an ROP from a repeater for which it already has prepared a label, it disregards the ROP.

15. Finite buffer size, b, i.e., the number of labels in the label queue cannot exceed b. Whenever $b \geq m$, this does not introduce any constraint on the initialization process.

8.3 A MARKOV CHAIN INITIALIZATION MODEL FOR A COMPLETE INTERFERENCE SYSTEM

This initialization process can be readily understood by analyzing the status of an arbitrary repeater as the process evolves. Specifically, at any time during the initialization process, a repeater will have one of the following statuses:

1. Transmitting ROP's, but has not successfully sent one to the station.

2. Transmitting ROP's, but has already successfully sent one to the station.

3. Received a label from station in previous slot, will forward ETE Ack to the station in next slot.

4. Awaiting another label from station. It has received at least one label from the station but the subsequent ETE Ack('s) were unsuccessful. In this case the repeater is not transmitting ROP's.

5. Completed initialization: repeater has received label from station and forwarded a successful acknowledgment.

The initialization process is begun with all repeaters having status 1 above and is completed when all repeaters have status 5. Figure 1 illustrates the relationship between the aforementioned statuses. As discussed above, a repeater begins the initialization process with status 1 during which the repeater is transmitting ROP's to the station. When one of these ROP's reaches the station,

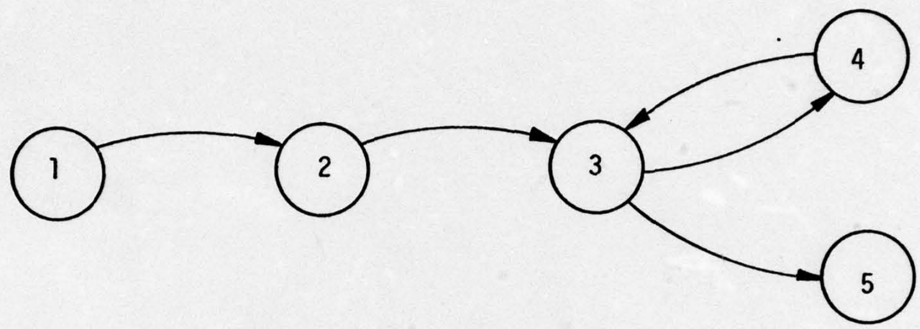

FIGURE 1: STATUSES OF A REPEATER DURING INITIALIZATION AND POSSIBLE TRANSITIONS FROM STATUS 1 (NON INITIALIZED) TO STATUS 5 (INITIALIZED)

the repeater acquires status 2, but it continues to transmit ROP's. The next change in status occurs when the station successfully sends a label to the repeater. At this time the repeater acquires status 3 and ceases transmitting ROP's. In the slot immediately following the slot in which it received the label, the repeater will forward an ETE Ack to the station. If this Ack is received by the station, the initialization of this repeater is complete and it acquires status 5. Otherwise, the repeater takes on status 4 where it awaits the receipt of another label. In this case the repeater will re-acquire status 3 when it receives another label and the action described above for status 3 will be repeated. Eventually the re-repeater will acquire status 5.

Note that since the station can forward only one label in any particular slot, at most one repeater can have status 3 at any given time. Also, the finite buffer size assumption requires that the total number of repeaters having statuses 2, 3, and 4 cannot exceed b.

8.3.1 Mathematical Formulation

In order to compute the expected time to initialize all repeaters, we introduce a Markov Chain based on the repeater statuses discussed above. Given the status of each repeater at a particular time during the initialization process, we define the state of the chain as the quintuple $(m_1, m_2, m_3, m_4, m_5)$, where m_i denotes the number of repeaters with status i at that time. From the previous section it is clear that m_i, i = 1, ..., 5 are integers satisfying

$$0 \leq m_i \leq m \quad , \quad i=1, \ldots, 5, \; i \neq 3.$$

$$0 \leq m_3 \leq 1$$

$$m_2 + m_3 + m_4 \leq b \quad \text{(buffer size constraint).}$$

Because of the assumptions of complete interference and zero capture, it is simple to compute the state transition matrix. First note that:

- For a packet to be successfully sent from a repeater to the station, all other repeaters and the station must be silent.

- For a packet to be successfully sent from the station to a repeater requires all repeaters to be silent.

Given the current state of the chain $(m_1, m_2, m_3, m_4, m_5)$, the following list enumerates all possible transitions. For convenience, the cases $m_3=0$ and $m_3=1$ are treated separately.

A. No repeater received a label in the previous slot ($m_3=0$).

1.a Successful ROP when station queue not empty. This requires $m_1 > 0$, $0 < m_2 + m_4 < b$. Transition to state $(m_1-1, m_2+1, 0, m_4, m_5)$ occurs with probability $(1-q)m_1p(1-p)^{m_1 + m_2-1} = z_1$.

1.b Successful ROP when station queue is empty. This requires $m_1 > 0$, $m_2 + m_4 = 0$. Transition to state $(m_1-1, m_2+1, 0, m_4, m_5)$ occurs with probability $m_1\ p(1-p)^{m_1 + m_2-1} = z_1$.

2. Successful label to a repeater with status 2. This requires $m_2 > 0$. Transition to state $(m_1, m_2-1, 1, m_4, m_5)$ occurs with probability $(1-p)^{m_1 + m_2} q\ m_2/(m_2+m_4) = z_2$.

3. Successful label to a repeater with status 4. This requires $m_4 > 0$. Transition to state $(m_1, m_2, 1, m_4-1, m_5)$ occurs with probability $(1-p)^{m_1+m_2} q\ m_4/(m_2+m_4) = z_3$.

4. Otherwise the status remains the same. This occurs with probability $1-z_1 - z_2 - z_3$.

B. A repeater received a label in the previous slot. $(m_3=1)$

1. Successful ETE Ack. Transition to state $(m_1,m_2,0,m_4,m_5+1)$ occurs with probability $(1-p)^{m_1 + m_2} (1-q)$.

2. Unsuccessful ETE Ack. Transition to state $(m_1,m_2,0,m_4+1,m_5)$ occurs with probability $1-(1-p)^{m_1+m_2} (1-q)$.

To facilitate our analysis, upon reaching the completely initialized state, (0,0,0,0,m), we restart the process by transferring to state (m,0,0,0,0) with probability 1. With this convention it is elementary to verify that:

- The Markov Chain is both aperiodic and irreducible.

- The initialization time is the interarrival times between visits to state (0,0,0,0,m) minus 1.

We can now employ the following well known theorem [CINLAR, 1975]:

Theorem: Let $\mathbb{P}$ be a state transition matrix for a finite, aperiodic, irreducible Markov Chain, then

a. There exists a stationary probability vector $\pi = (\pi_1, \ldots \pi_n)^T$ satisfying

$$\pi^T \mathbb{P} = \pi^T \tag{1}$$

$$\pi > 0 \tag{2}$$

$$\sum_{i=1}^{n} \pi_i = 1 \tag{3}$$

b. $^1/\pi_i$ is expected interarrival time between visits to state i.

From the discussion above, it is clear that this theorem can be applied to the Markov Chain defined earlier. Specifically, if (0,0,0,0,m) is the n^{th} state in the chain, then the expected initialization time

$$T_I = {}^1/\pi_n - 1. \tag{4}$$

Computation of the stationary vector, π, involves solving the linear equations

$$(\mathbb{P}-I)^T \pi = 0 \tag{5}$$

$$\sum_{i=1}^{n} \pi_i = 1 \tag{6}$$

These are n+1 equations with n unknowns π_i. The nxn matrix $(\mathbb{P}-I)^T$ has rank n-1, and any one of its equations may be dropped to form a full rank system. The solution can then be obtained by Gaussian elimination. When m is large, however, it may be necessary to employ sparse matrix techniques in the Gaussian elimination.

8.3.2 Number of States in the Chain

Since linear equations must be solved to determine the expected initialization time, it is important that the number of states not grow too rapidly as the number of repeaters increase. We consider two cases,

- Infinite buffer b=m, (b=m is equivalent to unlimited buffer size since more are not needed for labeling m repeaters).

- Fixed buffer, b<m, constant as m increases.

The problem of determining the number of states is equivalent to determining how many different ways m identical elements can be put into 5 different sets, G_i, i=1, ..., m with the proviso one set, say G_1, can have at most one element in it. Note the set G_1 corresponds to status 3 in the Markov Chain. We solve the problem iteratively. First let $m-k_1$ elements be put in sets G_1, G_2, and G_3, then the remaining k_1 elements are partitioned into the sets G_4 and G_5 in k_1+1 different ways. Now suppose $m-k_2$ elements are partitioned into G_1 and G_2; then the remaining k_2 elements can be partitioned so k_2-k_1 are in G_3 and k_1 are in G_4 and G_5. Applying the result for when the elements in G_1, G_2, and G_3 were fixed and summing, we now find total number of combinations is

$$\sum_{k_1=0}^{k_2} (k_1+1) = \frac{(k_2+2)(k_2+1)}{2} \tag{7}$$

Now consider two possible cases for G_1, since it can contain at most 1 element. Suppose $G_1 = \phi$, and that G_2 contains $m-k_2$ elements. Applying equation (7) and summing over allowable k_2, we obtain

$$\sum_{k_2=0}^{m} \frac{(k_2+2)(k_2+1)}{2} \tag{8}$$

Similarly when G_1 contains 1 element and G_2 contains $m-1-k_2$, we obtain

$$\sum_{k_2=0}^{m-1} \frac{(k_2+2)(k_2+1)}{2} \tag{9}$$

The total number of combinations is just the sum of (8) and (9), namely:

$$\sum_{k=0}^{m-1} (k+2)(k+1) + \frac{(m+2)(m+1)}{2} \tag{10}$$

The corresponding values in (10) for m=3, ..., 10 are given in Table 1.

Our numerical results show that to compute the expected initialization time using a PDP-10 on ARPANET requires approximately:

- 3 CPU seconds for a PR network with 3 repeaters and 3 buffers.

- 10 CPU seconds for a PR network with 4 repeaters and 4 buffers.

- 34 CPU seconds for a PR network with 5 repeaters and 5 buffers.

In these computations, we did not utilize the sparsity of the transition matrix. To compute initialization times for PR networks with more repeaters it would be highly advantageous to employ sparse matrix methods.

Now consider the case where buffer size is fixed equal to b and the number of repeaters m increases. We want to determine the asymptotic behavior of the number of states in the Markov Chain. Again this can be analyzed in terms of partitioning m identical elements into 5 sets with the provisos:

- One set, G_1, can have at most 1 element.

- The total number of elements in sets G_1, G_2, and G_3 is less than b.

To compute the number of states, the above results can be used. Suppose there are ℓ elements in G_1, G_2, and G_3 and $m-\ell$ elements in sets G_4 and G_5. Reasoning as above, we can show that there are

Number of Repeaters m	Number of States
3	30
4	55
5	91
6	138
7	204
8	290
9	395
10	510

TABLE 1: NUMBER OF STATES IN THE MARKOV CHAIN AS A FUNCTION OF NUMBER OF REPEATERS FOR b=m

$2\ell+1$ ways in which the ℓ elements can be partitioned in the sets G_1, G_2, and G_3; and $m-\ell+1$ ways the $m-\ell$ elements can be partitioned in the sets G_4 and G_5. Thus given ℓ there are $(2\ell+1)(m-\ell+1)$ ways of partitioning. Summing over all permissible ℓ, we determine the total number of states:

$$\sum_{\ell=0}^{b} (2\ell+1)(m-\ell+1) \tag{11}$$

However, since $0 \leq \ell \leq b$, we see

$$\sum_{\ell=0}^{b} (2\ell+1)(m-\ell+1) \leq \sum_{\ell=0}^{b} (2b+1)(m+1) \leq (2b+1)(b+1)(m+1) \tag{12}$$

Thus as m increases and b stays fixed, the number of states is bounded by a linear function of m. This result indicates that if we restrict the buffer size, we will be able to compute initialization times for large PR networks.

8.3.3 Computational Results

The formulae obtained via the Markov Chain initialization model were programmed and the initialization time of radio networks was studied as a function of number of repeaters, number of station buffers and the transmission rates by repeaters and stations.

In the first series of experiments we studied the systems with $m=3$, 4, and 5, and $b=m$. The repeater transmission rate, p, was systematically varied to approximate minimal initialization time. These results are tabulated in Table 2 and illustrated in Figures 1, 2, and 3, where the initialization times are plotted as a function of p for various values of station transmission rates q. Boxed entries in Table 2 indicate minimal values.

		m=3 b=3	m=4 b=4	m=5 b=5
q=.3	p=.05	T_I = 51.93	T_I = 61.85	T_I = 71.17
	.10	36.44	46.19	56.77
	.15	33.67	45.68	60.03
	.20	34.43	49.76	69.17
	.25	36.98	56.57	82.54
	.30	40.86	65.95	100.84
q=.4	p=.05	52.09	62.36	72.03
	.10	35.96	45.53	$\boxed{55.68}$
	.15	32.44	$\boxed{43.56}$	56.44
	.20	$\boxed{32.37}$	45.99	62.75
	.25	33.97	50.87	72.71
	.30	36.76	57.88	86.58
q=.5	p=.05	53.48	64.48	74.93
	.10	37.11	47.28	58.00
	.15	33.27	44.69	57.65
	.20	32.80	46.29	62.42
	.25	33.90	50.08	70.37
	.30	36.07	55.69	81.51
q=.6	p=.05	56.23	68.42	80.15
	.10	37.11	51.34	63.49
	.15	35.99	48.68	62.96
	.20	35.39	50.000	67.09
	.25	36.28	53.27	73.92
	.30	38.12	58.07	83.43

TABLE 2: STUDY ON REPEATER RATE AND STATION RATE FOR b=m

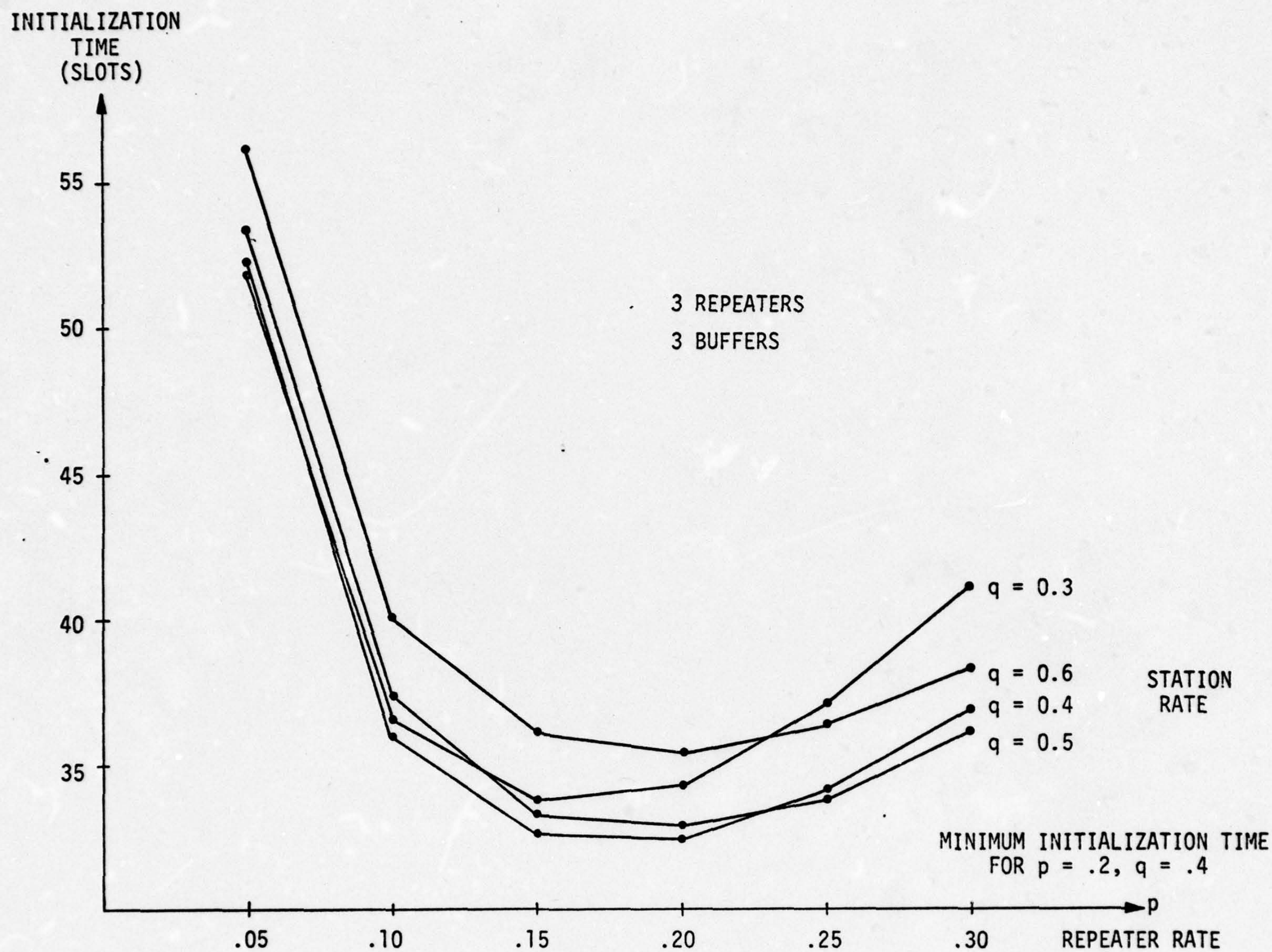

FIGURE 1: INITIALIZATION TIME AS A FUNCTION OF REPEATER TRANSMISSION RATE FOR m=b=3

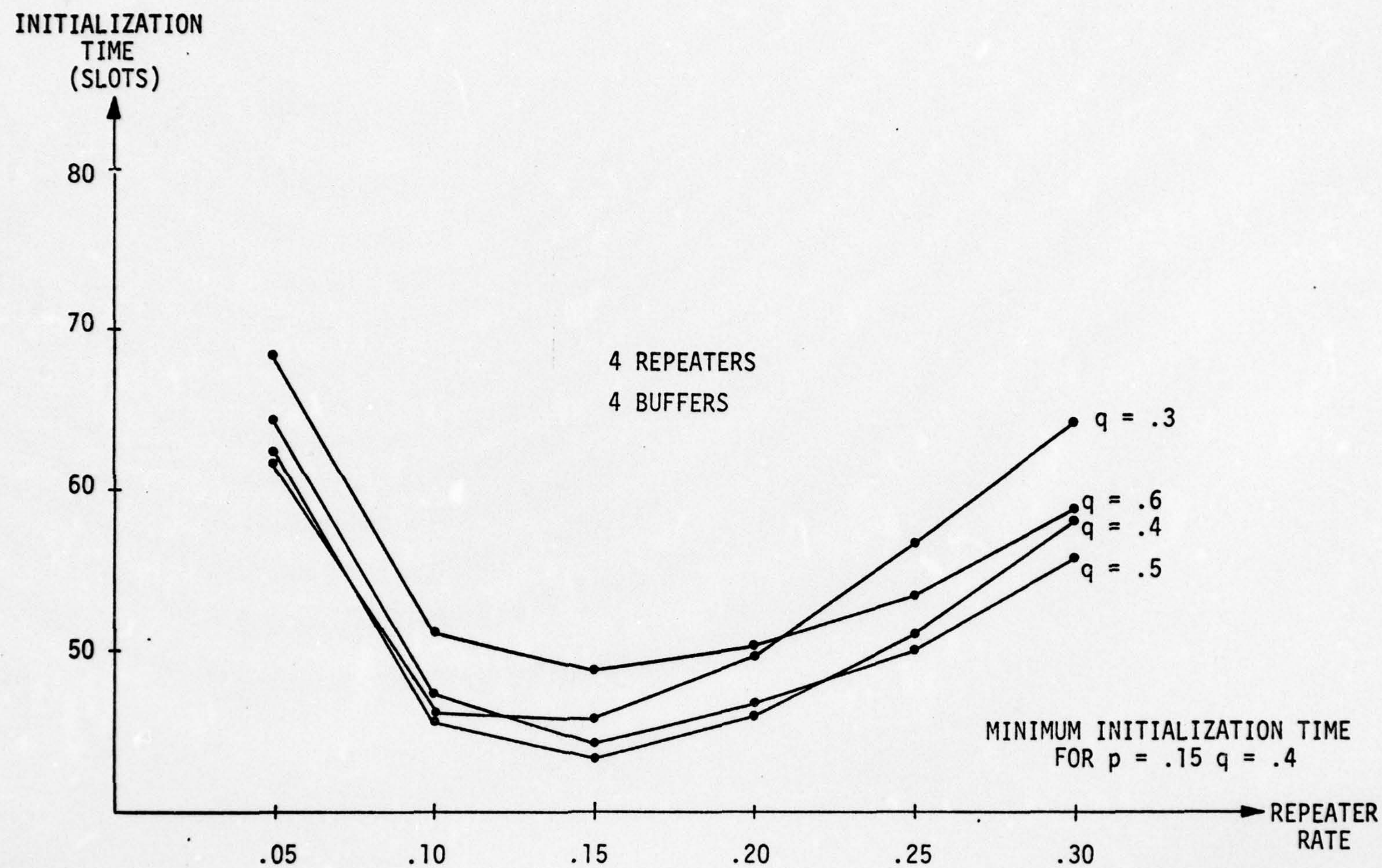

FIGURE 2: INITIALIZATION TIME AS A FUNCTION OF REPEATER TRANSMISSION RATE FOR m=b=4

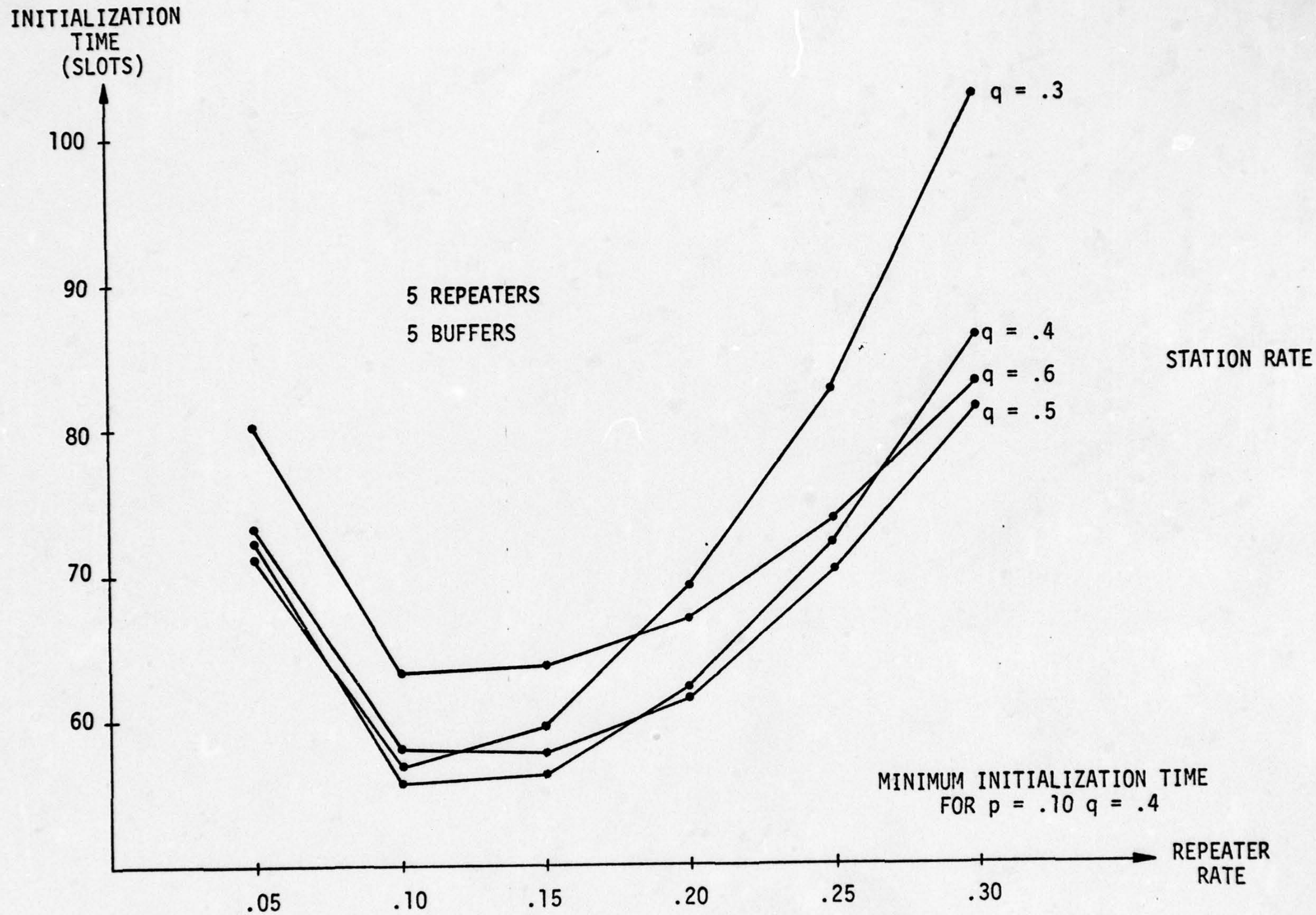

FIGURE 3: INITIALIZATION TIME AS A FUNCTION OF REPEATER TRANSMISSION RATE FOR m=b=5

First note the convex shape of the curves in these figures. We conjecture that this convexity in p holds for all values of q, m, and b but have not endeavored to prove it. Because of this convexity and the continuity of the initialization times as a function of p and q (for physical reasons), these curves give a good indication of the optimal values of p and q.

From each of these figures it is clear that if a poor repeater rate is selected (e.g., p=.05) the initialization time will be high regardless of what the station rate q is. However, if a poor station rate is chosen (e.g., q=0.6) but a correspondingly good repeater rate is chosen, the effect on initialization time will be relatively small. Thus we conclude that the initialization is more sensitive to the repeater transmission rate of ROP's than the station transmission rate of labels, for the range of p and q investigated, i.e., values near the optima.

Comparing Figure 1 through 3, we note that as the number of repeaters increases; the optimal values of repeater rate decreases while the optimal station rate remains the same. Realistically, we feel the optimal *station rate is not independent* of m, but that the courseness of our systematic search prevents us from detecting the sensitivity of the initialization time to the station rate. However, we feel it is valid to conclude that as the number of repeaters increase, it is more important to optimally adjust the repeater rate than the station rate.

It is interesting to note that the combined repeater rate (i.e., the total transmission rate of all repeaters) mp for optimal values of p remains relatively constant. Specifically, we obtained:

m = 3;	mp = (3).2 = .6
m = 4;	mp = (4).15 = .6
m = 5;	mp = (5).10 = .5

Since m is an integer, it is necessary to carry out a more refined systematic search for a larger number of repeaters to detect the sensitivity of the combined repeater rate to the number of repeaters.

To analyze the effect of a finite buffer, we have systematically varied p and q for a 5 repeater network with the number of buffers b = 1, 2, 3, 4, and 5. The entries in Table 3 are the resulting minimal initialization times, and Figure 4 clearly indicates the expected decreasing returns to scale property of additional buffers. This implies that if b is somewhat smaller than m, the initialization time is not appreciably affected. One way of interpreting this result is that when a 5 repeater system has 4 or 5 buffers, only in rare instances will the station have 4 or 5 labels in the label queue. Alternatively, when the station only has 3 buffers, only in rare instances will an ROP not be accepted because the label queue is full.

Choosing the number of buffers equal to approximately one-half the number of repeaters, we systematically varied p and q to determine the minimal initialization time as a function of m. These results are listed in Table 4 and illustrated in Figure 5. Because m and b are integers, it is not possible that b/m be exactly one-half in all cases. Surprisingly, however, the initialization times increase nearly linearly over the range of m considered.

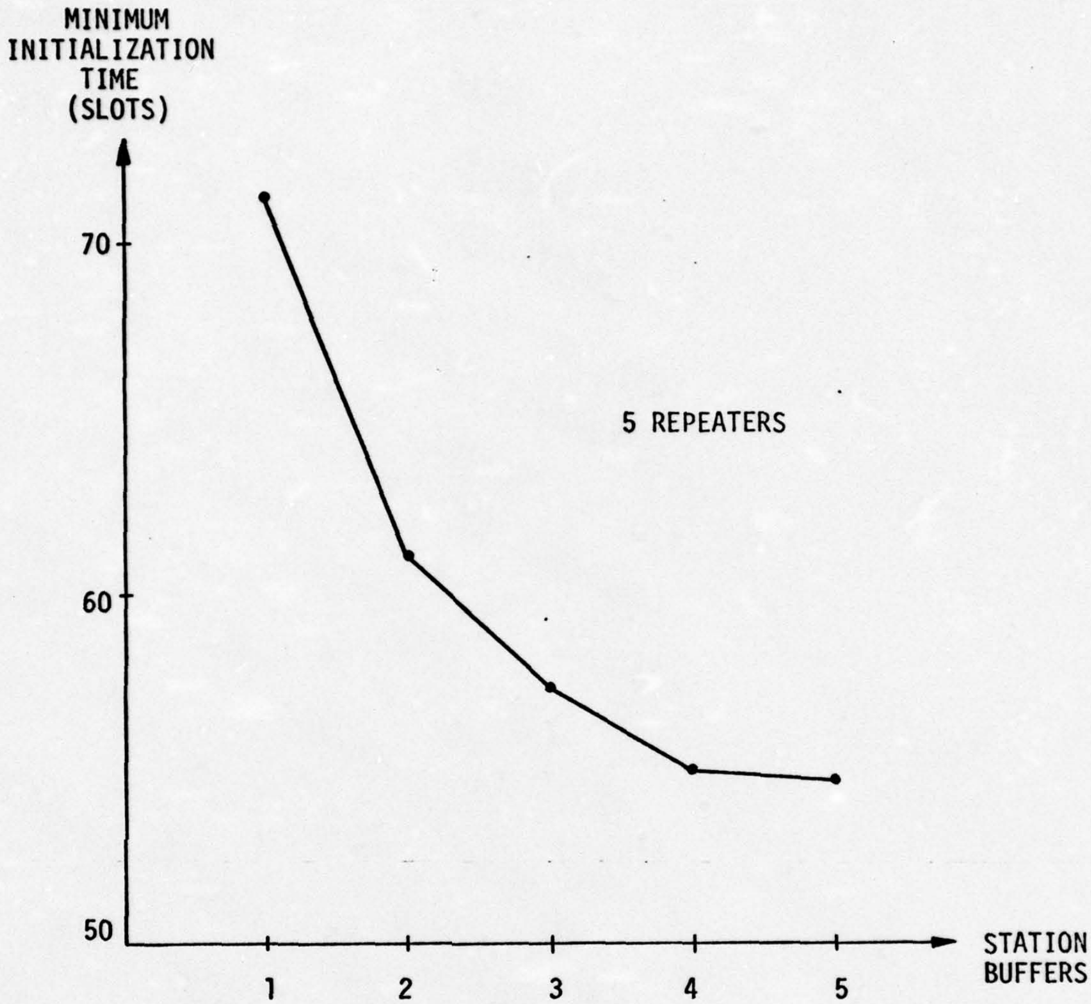

FIGURE 4: MINIMUM INITIALIZATION TIME AS A FUNCION OF NUMBER OF STATION BUFFERS

m=5	b=1	T_I = 71.21
	=2	= 61.15
	=3	= 57.45
	=4	= 55.99
	=5	= 55.68

TABLE 3: STUDY ON BUFFER SIZE FOR 5 REPEATERS

m=3	b=2	T_i = 33.89	T_I/m = 11.3	$T_I\left(b/m\right)$ = 22.6
m=4	b=2	47.27	= 11.8	= 23.6
m=5	b=3	57.45	= 11.5	= 34.5
m=6	b=3	70.85	= 11.8	= 35.4
m=7	b=3	86.10	= 12.3	= 36.9

TABLE 4: STUDY ON BUFFER SIZE FOR $b \approx m/2$

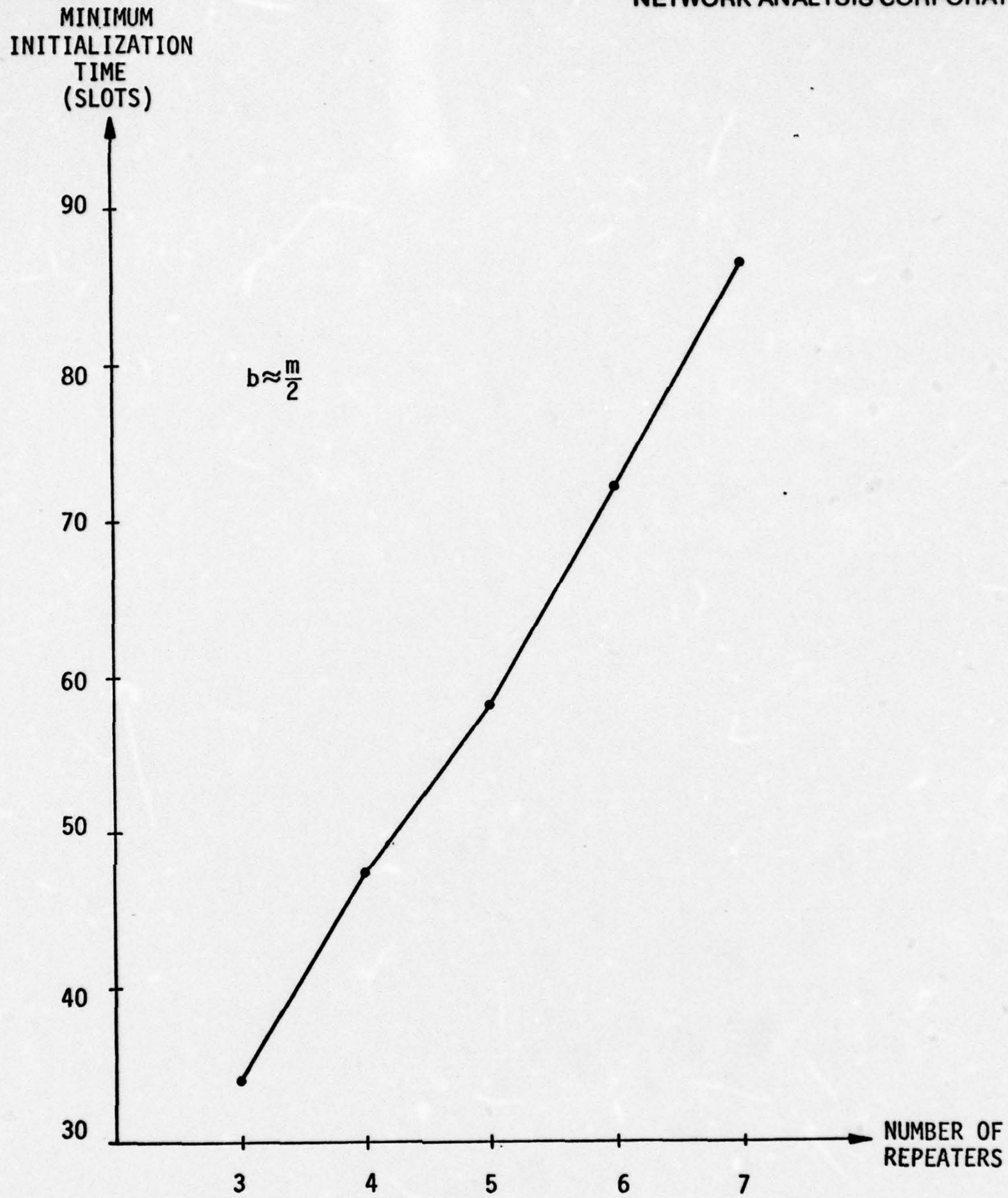

FIGURE 5: MINIMUM INITIALIZATION TIME AS A FUNCTION OF NUMBER OF REPEATERS

8.4. A MARKOV CHAIN INITIALIZATION MODEL FOR A SYSTEM WITH GENERAL INTERFERENCE PATTERN

8.4.1 Introduction

In the previous section a Markov Chain model for the initialization of 1-hop PRNET with complete interference has been presented and analyzed. The model assumed that once a device is broadcasting, every other device is blocked. This may not (and in fact, should not) occur in a real network where the repeaters are at a considerable distance from each other. In this section we extend the previous model by considering general interference patterns.

In the general interference environment, we need detailed information as to which repeaters have already been initialized, and are thus quiet, consequently we can expect a higher number of states in the present model than in the previous model. Hence, the computational complexity will increase.

8.4.2 Mathematical Formulation

The assumptions which define the model are the same as in Section 2 with the following modifications:

1. To each repeater R_i, there corresponds a set D_i of interfering repeaters, with $R_i \in D_i$, $S \notin D_i$.

2. When sending ROP's, repeater R_i is broadcasting into a given slot with probability p_i.

3. Infinite station buffer size for labels.

Let $R = \{R_j | j = 1,2,...m\}$ denote the given set of repeaters.

Let $\rho(R)$ be the power set of R, the set of all subsets of R.

Let $G_i \in \rho(R)$ be the set of repeaters which at a given slot are in status i, $1 \leq i \leq 5$. We write

$$G_i = \{R_{j_1}, R_{j_2}, ..., R_{j_{k(i)}}\} \tag{13}$$

Let $|G_i| = m_i$ be the cardinality of set G_i.

Let $\sigma = \rho(R) \times \rho(R) \times \rho(R) \times \rho(R) \times \rho(R) = \rho^5(R)$
Consider $\$ \subset \sigma$ with

$$\$ = \{(G_1,G_2,G_3,G_4,G_5),\ |G_3| \leq 1,\ G_i \cap G_j = \phi, \text{ for } 1 \leq i,j \leq 5,\ \bigcup_{i=1}^{5} G_i = R\} \tag{14}$$

Finally define a Markov Chain whose states are the points of \$ and transition probabilities are to be discussed.

Comments:

1. σ represents all quintuples (G_1,G_2,G_3,G_4,G_5) whose entries are subsets of R.

2. \$ represents all allowable quintuples in reference to the above assumptions. For example, we can only have one repeater in status 3, as already pointed out in the previous model.

3. $\eta = |\$|$ represents the number of states in the Markov Chain. We derive later an exact expression for η.

4. If $D_i = R$ for all i, this model reduces to the previous one.

The following discussion addresses the computation of the state transition matrix. Consider the possible transitions from the state $\$_L = (G_1,G_2,G_3,G_4,G_5)$, with L representing the time slot.

<u>Case 1. $G_3 = \phi$</u>

The following transitions are possible.

1. This transition represents successful reception of a ROP. Any one of $m_1 = |G_1|$ states (when $m_1 \neq 0$) are reached:

$$\$_{L+1} = (G_1-\{R_{j*}\},\ G_2 \cup \{R_{j*}\},\ \phi,\ G_4,\ G_5),\ \text{for } R_{j*} \in G_1 \tag{15}$$

with probability

$$z^1_{j*} = \begin{cases} p_{j*} \prod\limits_{R_v \in G_1 \cup G_2 - \{R_{j*}\}} (1-p_v)(1-q) & , \text{ if } m_2 + m_4 > 0 \\ \\ p_{j*} \prod\limits_{R_v \in G_1 \cup G_2 - \{R_{j*}\}} (1-p_v) & , \text{ if } m_2 + m_4 = 0 \end{cases} \tag{16}$$

If $p_x = p$ for all x (henceforth called "normality") we obtain:

$$z_{j*}^{1} = \begin{cases} p(1-p)^{m_1 + m_2 - 1}(1-q) & , \quad \text{if } m_2 + m_4 > 0 \\ p(1-p)^{m_1 + m_2 - 1} & , \quad \text{if } m_2 + m_4 = 0 \end{cases} \tag{17}$$

2. This transition represents successful reception of a label. Any one of $m_2 = |G_2|$ states (when $m_2 \neq 0$) are reached:

$$\$_{L+1} = (G_1, G_2 - \{R_{j*}\}, R_{j*}, G_4, G_5) \quad , \quad \text{for } R_{j*} \in G_2 \tag{18}$$

with probability

$$z_{j*}^{2} = \prod_{R_v \in G_1 \cup G_2 \cap D_{R_{j*}}} (1-p_v) \frac{q}{m_2 + m_4} \tag{19}$$

(observe that $m_2 + m_4 > 0$, since $m_2 > 0$)

Under normality,

$$z_{j*}^{2} = (1-p)^{|G_1 \cup G_2 \cap D_{R_{j*}}|} \frac{q}{m_2 + m_4} \tag{20}$$

3. This transition represents successful re-reception of a label. Any one of $m_4 = |G_4|$ states (when $m_4 \neq 0$) are reached:

$$\$_{L+1} = (G_1, G_2, \{R_{j*}\}, G_4 - \{R_{j*}\}, G_5) \quad \text{for } R_{j*} \in G_4 \tag{21}$$

with probability

$$z_{j*}^{4} = \prod_{R_v \in G_1 \cap G_2 \cap D_{R_{j*}}} (1-p_v) \frac{q}{m_2 + m_4} \tag{22}$$

Under normality,

$$z^4_{j*} = (1-p)^{|G_1 \cup G_2 \cap D_{R_{j*}}|} \tag{23}$$

4. Remain in same state with probability

$$1 - \sum_{R_j \varepsilon G_1} z^1_j - \sum_{R_j \varepsilon G_2} z^2_j - \sum_{R_j \varepsilon G_4} z^4_j \tag{24}$$

where a summation over the empty set is 0.

<u>Case 2. $G_3 = \{R_j\}$</u>

1. This transition represents successful ETE reception. Transition is to state $\$_{L+1} = (G_1, G_2, \phi, G_4, G_5 \cup \{R_j\})$

   with probability:

$$r = \prod_{R_v \varepsilon G_1 \cup G_2} (1-p_v)(1-q) \tag{25}$$

   under normality,

$$r = (1-p)^{m_1+m_2}(1-q). \tag{26}$$

2. This transition represents unsuccessful ETE Ack. Transition is to state

   $\$_{L+1} = (G_1, G_2, \phi; G_4 \cup \{R_j\}, G_5)$ with probability 1-r.

Case 3. Transition back to initial state

1. Consider $\$_L = (\phi,\phi,\phi,\phi, R)$. We assume that transition occurs to state $\$_1 = \{R,\phi,\phi,\phi,\phi\}$ with probability 1.

Comments

1. Figure 6 depicts the partial state diagram for a 3-repeater example.

2. Observe the divergence from the model in Section 3 at Case 1.2 and 1.3. The latter model is recaptured by taking $D_i = R$ for all i. Then, in order to have a transition from (G_1,G_2,ϕ,G_4,G_5) to $(G_1,G_2 - \{R_{j*}\}, \{R_{j*}\}, G_4, G_5)$, for $R_{j*} \varepsilon G_2$, namely, a label is successfully received by R_{j*}, we require that every unlabeled repeater is silent, which occurs with probability $((1-p)^{|G_1 \cup G_2|})$. In the present model we only require that every unlabeled repeater which interferes with R_{j*} is silent. (Thus, the probability is $(1-p)^{|G_1 \cup G_2 \cap D_{j*}|}$). Similarly for case 1.3.

3. A finite buffer size constraint is easily added to the model by readjusting the transition probabilities to 0 when the buffer is full.

$D_1 = \{R_1, R_2\}$

$D_2 = \{R_1, R_2, R_3\}$

$D_3 = \{R_1, R_3\}$

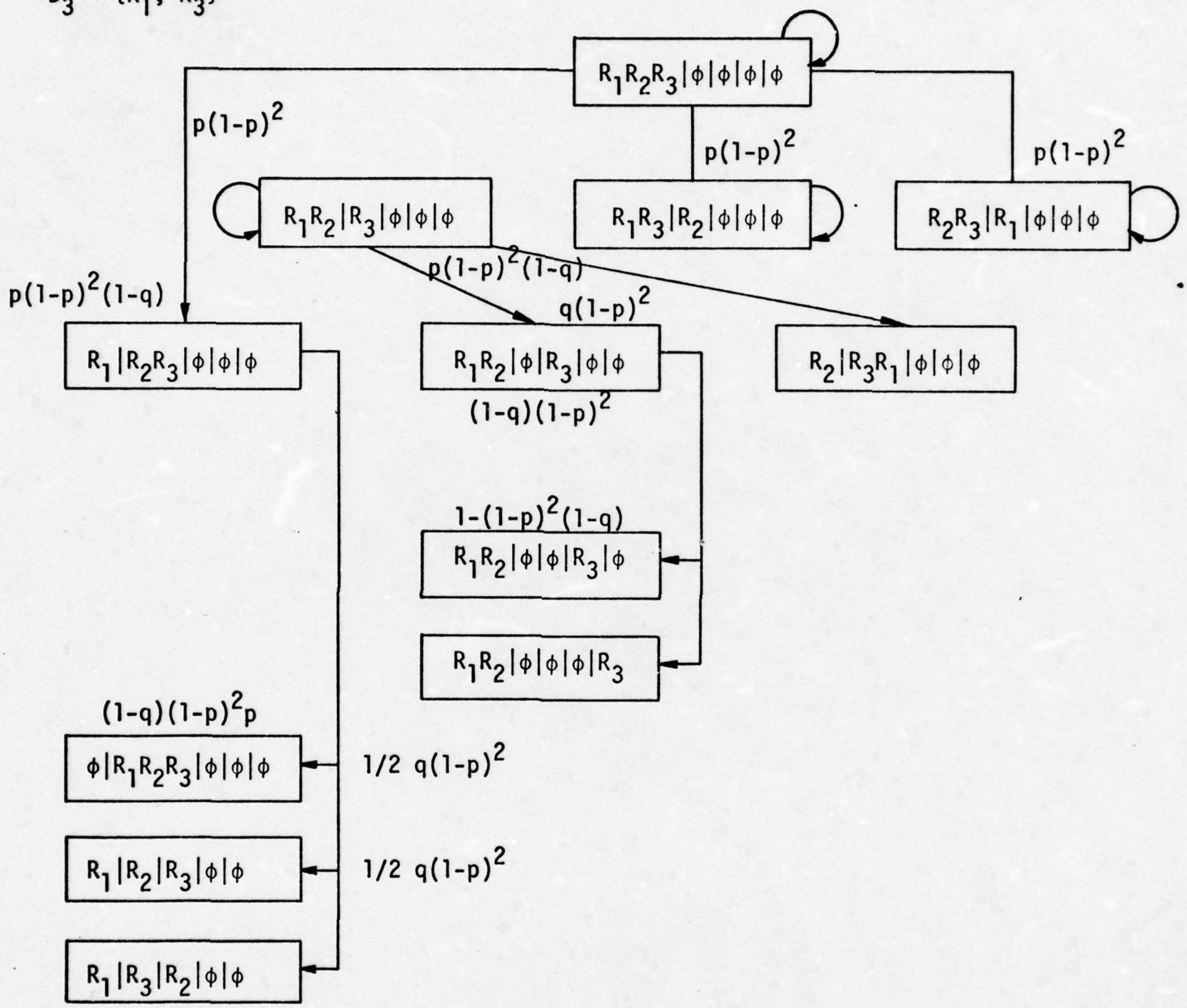

FIGURE 6: A PARTIAL STATE DIAGRAM FOR A 3-REPEATER NETWORK

8.4.3 Calculation of Number of States

The cardinality of $ is derived below in terms of number theoretic quantities.

Let:

a. $|R| = m$

b. ξ_m be the number of partitions of m elements into exactly 5 groups.

c. ${}_LP_i(m)$, the i-th component of the L-th partition, is the number of elements in the i-th group ordered as

$$ {}_LP_{i+1}(m) \leq {}_LP_i(m) \quad , \quad 1 \leq i \leq 5, \tag{27} $$

with ${}_LP_i(m) = 0$ if necessary.

d. ${}_LK_j(m)$ be the number of groups in the L-th partition of m elements having j elements.

From these definitions it follows that

a. $$ m = \sum_{i=1}^{5} {}_LP_i(m) \quad , \quad 1 \leq L \leq \xi_m \tag{28} $$

b. $$ m = \sum_{j=0}^{m} j\, {}_LK_j(m) \tag{29} $$

Let

$$\Phi_{{}_LP_i(m)}(b) = \begin{cases} b & \text{if } {}_LP_i(m) \leq 1 \\ b-1 & \text{if } {}_LP_i(m) > 1 \end{cases} \tag{30}$$

We then claim that *the number* of *states in the Markov Chain* is given by:

$$|\$| = \sum_{L=1}^{\xi_m} \frac{\displaystyle\prod_{v=0}^{4} \binom{m - \sum_{i=1}^{v} {}_LP_i(m)}{{}_LP_{v+1}(m)}}{\displaystyle\prod_{j=0}^{m} {}_LK_j\,!} \prod_{i=0}^{4} \Phi_{{}_LP_i}(5-i) \tag{31}$$

Proof: Consider a given partition L, given by ${}_LP_i(m)$, i=1, ...,5. The numerator term involving the product of the binomials represents the number of ways we can partition m elements (repeaters) into groups G_{v+1} *having* ${}_LP_{v+1}(m)$ elements, v + 1 = 1, 2, ..., 5, with duplication, i.e., two groups may have same number of elements. The denominator term eliminates the duplicate copies due to permutation, embodied in the partition operation of the numerator, i.e., due to two groups having same number of elements. Finally the product term represents how the distinct partition groups G_i, $1 \leq i \leq 5$, can be assigned to a particular status; if a set has more than one element it can only be placed into any of 5-i-1 statuses, where $1 \leq i \leq 5$ represents the statuses already filled up; if the set has one or no element in it, it can be placed into any of 5-i statuses. Condensed mathematically, any set can be placed into $\Phi_{{}_LP_i}(5-i)$ statuses. The above restriction is imposed by the fact that at most one repeater can be in status 3, during any ALOHA slot.

Computations show that:

$$|\$(3)| = 112$$
$$|\$(4)| = 488$$
$$|\$(5)| = 2320$$

The solution strategy should follow the procedure used for the model in Section 3.

8.5 CONCLUSIONS

Markov Chain initialization models for a single hop packet radio network were developed; network initialization time was studied as a function of number of repeaters, number of station buffers, transmission rates of ROP's by repeaters, and transmission rate of labels by the station. The models appear to be complex from the computational viewpoint; namely, the number of states of the chain is of the order $0(m^3)$, m being the number of repeaters (the complexities of the models were derived). The high complexity enabled the numerical investigation of small networks only of up to 7 repeaters, and this could be done for one of the models only, which assumed a complete interference system in which all repeaters and the station are within an effective transmission range of each other.

The computational results suggest the following conclusions:

- Initialization times are more sensitive to repeater transmission rates than station rates.

- Initialization times are not significantly increased if there are somewhat fewer buffers in the station (for initialization) than the number of repeaters.

- Initialization times increase nearly linearly as the number of repeaters increases (with the number of buffers in the station equal to approximately 1/2 the number of repeaters).

- Optimal repeater transmission rates decrease as the number of repeaters increase.

- The total transmission rate of all repeaters is nearly constant.

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CHAPTER 9

MARKOV CHAIN INITIALIZATION MODELS WITH FIFO LABEL QUEUE MANAGEMENT AT THE STATION

CHAPTER 9

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A MARKOV CHAIN INITIALIZATION MODEL WITH FIFO LABEL QUEUE MANAGEMENT AT THE STATION

9.1 INTRODUCTION

We address the problem of intialization of repeaters in packet radio networks. Repeaters in the radio network need to be initialized by the station; namely, to receive a so-called "label" which enables addressing specific devices during packet transportation. It is assumed that a priori the station is not informed of the existence and location of specific repeaters. Hence, the initialization process involves sending special control packets, called Repeater On Packets (ROP's) from repeaters to the station to inform existence and location, the transmission of labels by the station to repeaters identified, and the transmission of End-to-End Acknowledgments (ETE Ack) by repeaters which receive a label. A repeater is considered by the station to be initialized after the station receives an ETE Ack to the label packet. The more interesting initialization problem is for multi-hop packet radio networks which will be the structure of practical systems. However, such systems are too complex and do not lend themselves to analysis. Nevertheless, the model and results for initialization of single-hop networks presented in this chapter provides much insight into the behavior of the initialization time and the proper transmission rates of repeaters and station to obtain minimum initialization time.

In Chapter 8 we discuss a Markov Chain model for initialization of 1-hop packet radio networks. This model is conceptually very useful, but in order to compute the total initialization time, it is necessary to solve a set of linear simultaneous equations. For an m repeater network the number of such equations turns out to be $O(m^3)$. Hence this model is not applicable for analyzing large networks. Also, only a model for complete interference is implemented.

The complexity of the model in Chapter 8 can be reduced by modifying the label queue management from a random selection discipline to a first in, first out (FIFO) discipline, the number of linear simultaneous equations that need be solved can be reduced from $O(m^3)$ to $O(m^2)$. In this chapter we:

1. Formulate a new Markov Chain initialization model based on FIFO queue management.

2. Establish that the complexity of this model is $O(m^2)$.

3. Derive exact solutions for this new model when the station has 1 or 2 buffers for the complete interference case.

4. Derive approximate solutions for this new model when the station has 1 or 2 buffers for the general interference case.

5. Study numerically the solution and obtain results for the proper transmission rates of station and repeaters to minimize the initialization time, as a function of number of repeaters, interference between repeaters and number of station buffers.

9.2 MODEL DEFINITION AND ASSUMPTIONS

In this analysis we make virtually the same assumptions of Chapter 8, except where noted. We are concerned with a PR network having the following characteristics:

1. Single station, S, and repeaters, R_j, $j=1, \ldots, m$.

2. Each repeater can communicate with the station (1 hop system).

3. Each repeater has I disturbing repeaters in addition to itself. When $I = m-1$, the following analyses are exact; otherwise approximations must be used. Note the computational results in Chapter 8 are for $I = m-1$.

4. No repeater accepts Repeater On Packets (ROP's) or End-To-End acknowledgements (ETE Ack's) from other repeaters.

5. Single channel slotted ALOHA access scheme.

6. When forwarding ROP's, each repeater broadcasts into a given slot with probability p.

7. If a repeater receives its label from the station, it forwards one ETE Ack into the next slot and halts transmission of ROP's.

8. After the station receives a ROP from a particular repeater for the first time, the station prepares a label for that repeater and places it in the label queue. We assume this requires zero processing time.

9. The station maintains the repeater label in the queue until it forwards the label to the repeater and receives an ETE Ack, indicating the label has been received. When this occurs, the repeater is considered labeled.

10. When the label queue is not empty, the station transmits a label with probability q.

11. When transmitting a label the station selects it from the queue on a FIFO basis. This assumption diverges from the equivalent assumption of Chapter 8, however for the case of 1 buffer the two assumptions agree.

12. Whenever the station receives a ROP from a repeater for which it already has prepared a label, it disregards the ROP.

13. Zero propagation time for radio signals.

14. Buffer size, b, i.e., the number of labels in the label queue cannot exceed b, b = 1, 2, ..., m.

15. If the station or a repeater receive more than one packet in the same slot, it is assumed that all packets are in error (zero capture).

It can be noted that at any time during the initialization process, a repeater will have one of the following statuses:

1. Transmitting ROP's, but has not successfully sent one to the station.

2. Transmitting ROP's, but has already successfully sent one to the station.

3. Received label from station in previous slot, will forward ETE Ack to station in next slot.

4. Awaiting another label from station. It has received at least one label from the station but the subsequent ETE Ack('s) were unsuccessful. In this case the repeater is not transmitting ROP's since it has a label. However, the station did not receive an ETE Ack to the label; hence it will keep on transmitting the label to the same repeater.

5. Completed initialization - repeater has received label from station and forwarded a successful acknowledgement.

As in Chapter 8 for the complete interference case, let m_i be the number of repeaters having status i, i=1, ..., 5, and define the Markov Chain whose state is the quintuple

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{pmatrix}$$

From the assumptions it is immediately clear that:

1. $\sum_{i=1}^{5} m_i = m$

2. $m_2 + m_3 + m_4 \leq b$

3. $m_3 + m_4 \leq 1$

Note that these are the same constraints used in the previous model, with the addition that $m_3 + m_4 \leq 1$ (implying that $m_4 \leq 1$). A repeater in status 3 has received a label in the previous slot; since the station can only send one label at a time, only one repeater could have such status; a repeater with status 4 is one that received a label but has not forwarded successfully an ETE Ack. The FIFO condition implies that the station keeps on sending a label to the same repeater, until it successfully receives an Ack; therefore only one repeater could be in status 4. Also the station could not pick a label for a repeater in status 2, if there is a backlogged repeater in status 4. Thus we need to impose $m_3 + m_4 \leq 1$.

Transition probabilities for this model with complete interference are derived in Appendix A. Analogous to the discussion in Chapter 8, this new model can be extended to treat the general interference case with no conceptual difficulties. The major complication is that because of the FIFO queue management, the model must note the order in which the station receives ROP's from the various repeaters. However, the computational complexity of the general interference model is again much greater than that of the complete interference model.

9.3 MAIN RESULTS - STUDY OF INITIALIZATION TIME FOR 1 AND 2 STATION BUFFERS FOR FIFO LABEL QUEUE SERVICE DISCIPLINE

In this section we present the closed form solutions for the initialization times. Furthermore, we study by numerical methods the optimum values of transmission rates by repeaters and station which result in minimum initialization time as a function of number of repeaters and the interference between repeaters. Finally in Section 9.3.5 we compare the initialization times for 1 and 2 station buffers.

9.3.1 Single Station Buffer

In Appendix A we show that the initialization time for a single hop m-repeater packet radio network is given by:

$$\bar{T} = \sum_{i=0}^{m-1} \left\{ \frac{1}{(m-i)p(1-p)^{m-i-1}} + \frac{1}{q(1-p)^{I(i)+1}} + \frac{1}{(1-q)(1-p)^{m-i-1}} \left(1 + \frac{[1-(1-q)(1-p)^{m-i-1}]}{q(1-p)^{I(i)}}\right) \right\} \quad (1)$$

Where:

p	-	Transmission rate of ROP's by an uninitialized repeater
q	-	Station label transmission rate
I(i)	-	Average number of interfering repeaters, given i rpeaters are initialized.

For complete interference the solution is exact, i.e., $I(i) = m-1-i$.

9.3.2 Numerical Results For The Single Station Buffer Case

The numerical study involved point-by-point search to obtain optimal parameter values accurate to two significant places. The major results are as follows:

Figure 1 depicts optimal values for the exact solution to the Markov Chain with complete interference. Observe that for complete interference the optimal station transmission rate is a constant $q^* = .44$. Note that this rate is higher than the optimal rate for the infinite buffer case and random service of label queue of Chapter 8. This is due to the fact that the station must clear its (only) buffer as soon as possible before it can proceed with the initialization.

Figure 2 depicts optimal values for the approximate non-complete interference. Observe that for partial interference q^* is still nearly constant, but there is a tendency to decrease as m increases. The optimal repeater transmission rate for complete interference case is lower than the optimal repeater rate for non-complete interference. In this case, a frequent repeater transmission blocks label reception, for any other repeater; to alleviate this, repeaters should not broadcast too frequently. On the other hand, for marginal interference, a repeater blocks only few other repeaters; thus we can afford a slightly higher repeater rate.

In Figures 1 and 2 we see that for both categories of interference $p^* < \frac{1}{m}$. Specifically,

$$I = m-1, \quad p^* \approx \frac{.7}{m} \text{ (m large)}$$

$$I = 2\ , \quad p^* \approx \frac{.8}{m} \text{ (m large)}$$

Also, the initialization time decreases as the interference decreases. For large m, there is approximately a 20% differential

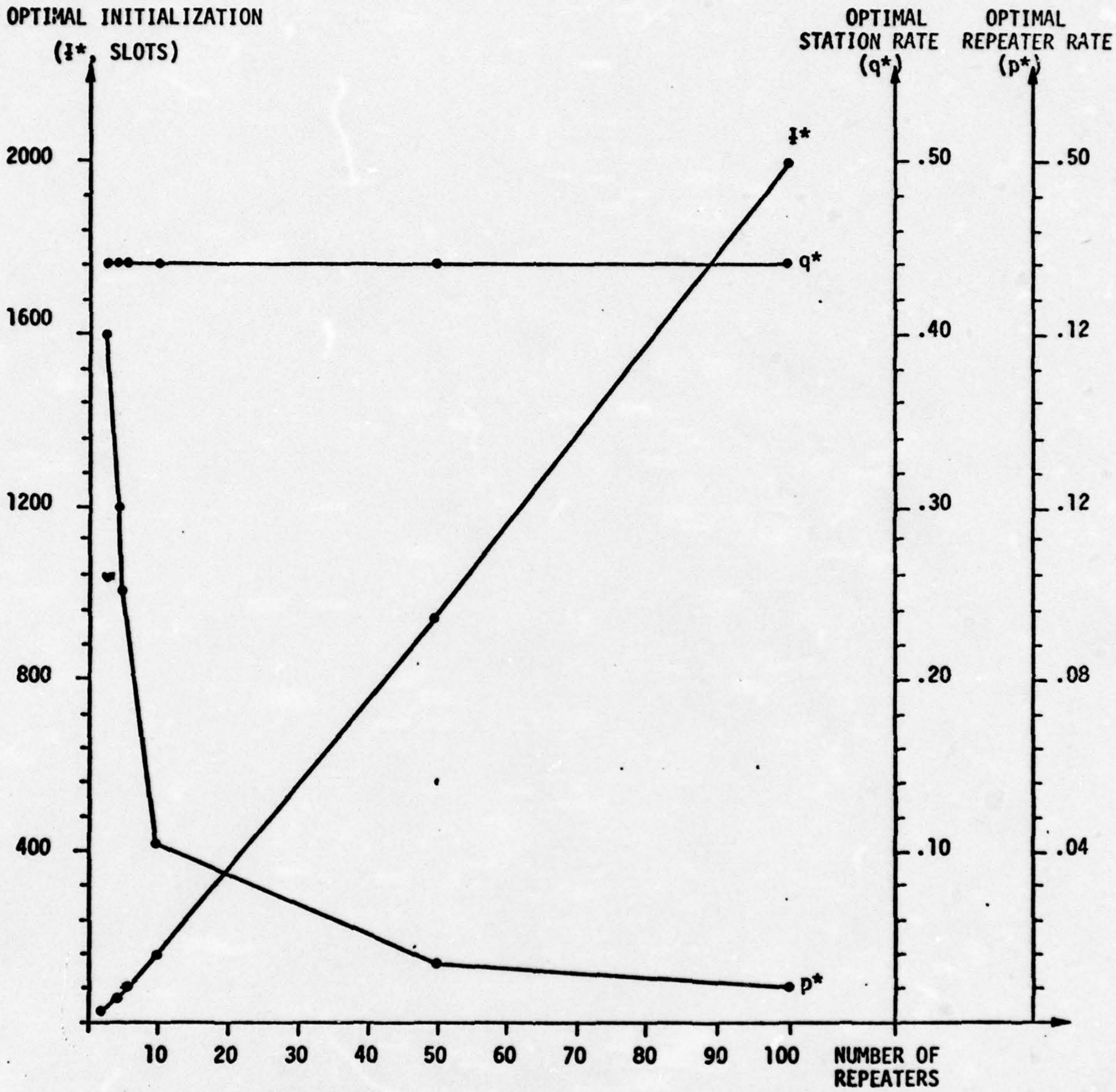


FIGURE 1: COMPLETE INTERFERENCE, ONE BUFFER

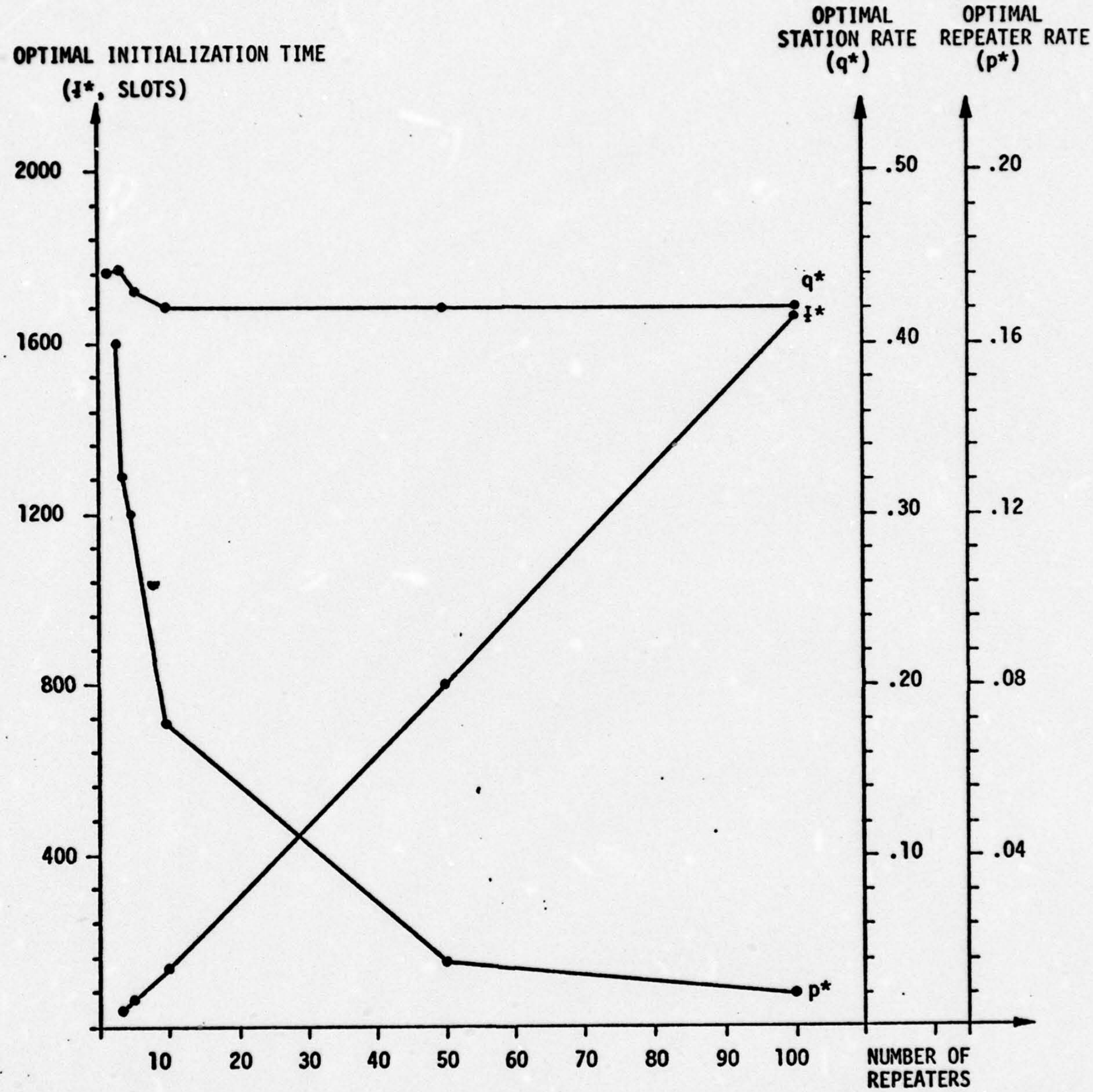


FIGURE 2: PARTIAL INTERFERENCE I=2, ONE BUFFER

between the case I=m-1 and I=2. Figure 3 shows the convex behavior and sensitivity of the initialization time as a function of the parameters.

Table 1 shows the time required for ROP collection and label delivery operation as a function of the cycle, indicating the relative difficulty as initialization progresses. A cycle is a section of the state transition diagram of the Markov Chain initialization model developed in Appendix A. Cycle i is the section of the diagram beginning with having i repeaters initialized and ending with having i+1 repeaters initialized. Figure 4 graphically illustrates the comparison. Note that ROP collection becomes consistently harder as initialization proceeds; label delivery becomes consistently easier as initialization proceeds. The label delivery becomes easier since there is less contention while the ROP collection becomes more difficult because there are few repeaters which produce acceptable ROP's. This is true even if the interference has decreased - indicating that the fact that there are fewer repeaters has a more marked effect.

9.3.3 Two Station Buffer Case With FIFO Service Management

The total single hop m repeater initialization time (derived in Appendix B) is given by:

$$I = \frac{1}{mp(1-p)^{m-1}} + \frac{1}{q(1-p)} + \frac{2}{1-q} + \sum_{i=0}^{m-2} E[\bar{I}_i] \qquad (2)$$

where

$$E[\bar{I}_i]^* = \frac{1}{p(m-i-1)(1-q)(1-p)^{m-i-1} + q(1-p)^{1+I(i)}} +$$

$$\frac{q(1-p)^{1+I(i)}}{p(m-i-1)(1-q)(1-p)^{m-i-1} + q(1-p)^{1+I(i)}} \cdot$$

*Equation continues on next page.

$$\{1 + \frac{(1-q)(1-p)^{m-i-1}}{(m-i-1)p(1-p)^{m-i-2}} + [1-(1-q)(1-p)^{m-i-1}]$$

$$[\frac{1}{(m-i-1)p(1-p)^{m-i-2}(1-q)+q(1-p)^{I(i)}}$$

$$+\frac{1}{(1-p)^{m-i-1}(1-q)} \cdot \frac{(m-i-1)p(1-p)^{m-i-2}(1-q)}{(m-i-1)p(1-p)^{m-i-2}(1-q)+q(1-p)^{I(i)}}$$

$$[1 + \frac{1}{q(1-p)^{I(i)}}]]\} \{1-[1-(1-q)(1-p)^{m-i+1}]$$

$$\frac{q(1-p)^{I(i)}}{(m-i-1)p(1-p)^{m-i-2}(1-q)+q(1-p)^{I(i)}}\}^{-1}$$

$$+ \frac{(m-i-1)p(1-q)(1-p)^{m-i-1}}{p(m-i-1)(1-q)(1-p)^{m-i-1}+q(1-p)^{1+I(i)}}$$

$$\{\frac{1}{q(1-p)^{1+I(i)}} + \frac{1}{(1-p)^{m-i-1}(1-q)}$$

$$[1 + [1-(1-p)^{m-i-1}(1-q)] \frac{1}{q(1-p)^{I(i)}}]\} \tag{3}$$

where:

p = Repeater transmission rate

q = Station transmission rate

I(i) = Average number of interfering repeaters, given i repeaters are initialized.

Again the expression is exact for complete interference.

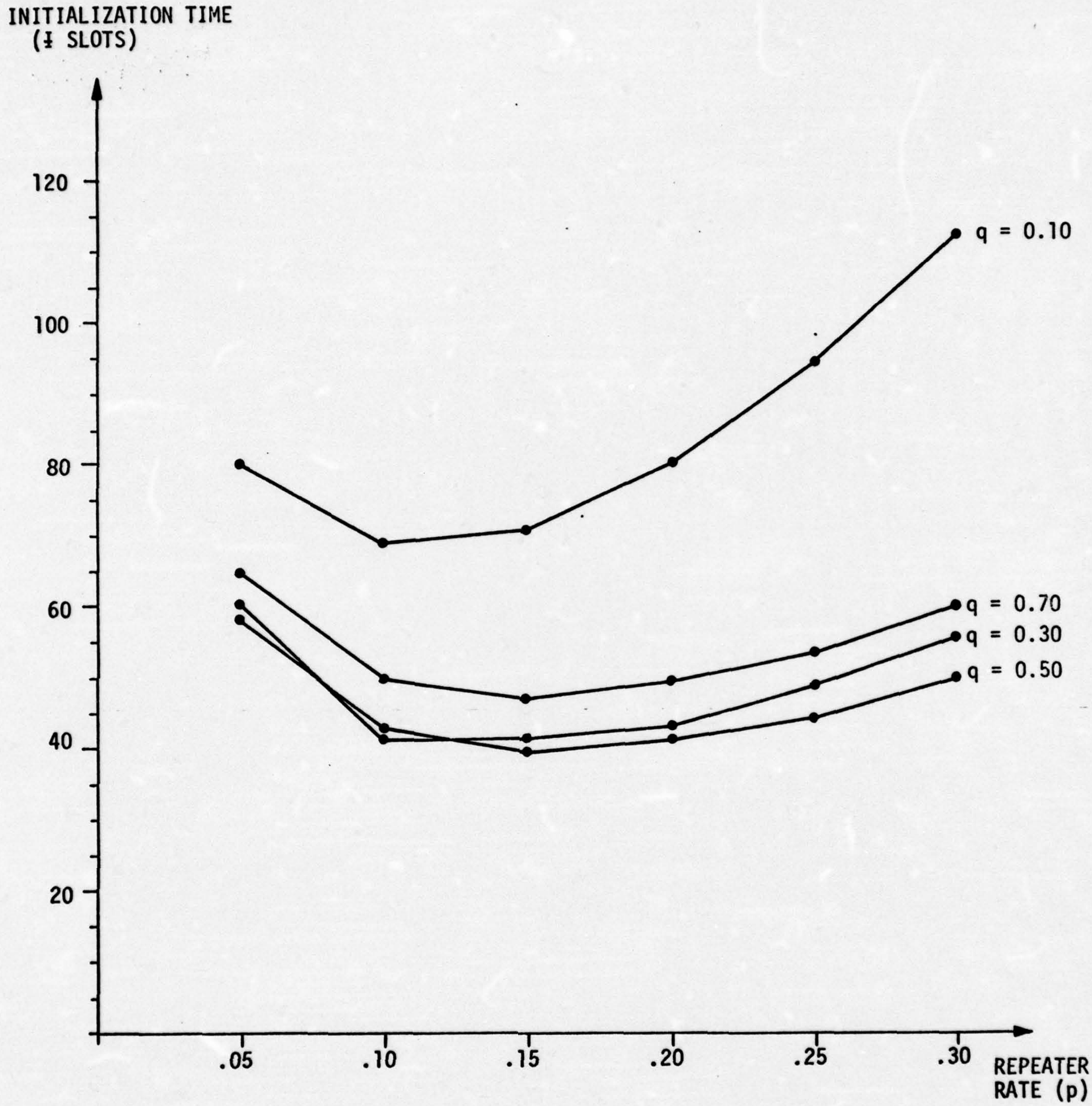


FIGURE 3: INITIALIZATION TIME, 3 REPEATERS, COMPLETE INTERFERENCE, ONE BUFFER

		i	1	2	3	4	5	6	7	8	9	10	TOTAL INITIALIZATION TIME (SLOTS)
m=10 q=.44 p=.15	I=9	$E_R^{(i)}$	2.8	2.7	2.5	2.5	2.5	2.5	2.7	3.0	3.9	6.6	Ŧ = 344.76
		$E_L^{(i)}$	85.0	62.6	46.3	34.3	25.5	19.0	14.3	10.8	8.1	6.2	
	I=2	$E_R^{(i)}$	2.8	2.7	2.5	2.5	2.5	2.5	2.7	3.0	3.9	6.6	Ŧ = 183.88
		$E_L^{(i)}$	32.5	26.7	21.3	18.0	14.8	12.2	10.1	8.3	6.9	5.7	
m=10 q=.44 p=.10	I=9	$E_R^{(i)}$	2.5	2.5	2.6	2.6	2.8	3.0	3.4	4.1	5.5	10	Ŧ = 198.71
		$E_L^{(i)}$	32	26	22	18	15	12.5	10.4	8.7	7.2	6.0	
	I=2	$E_R^{(i)}$	2.5	2.5	2.6	2.6	2.8	3.0	3.4	4.1	5.5	10	Ŧ = 147
		$E_L^{(i)}$	17	15	13	12	10	9.5	8.3	7.4	6.5	5.7	
m=10 I=9 q=.44 p=.05		$E_R^{(i)}$	3.1	3.3	3.5	3.8	4.3	4.9	5.8	7.3	10.5	20	Ŧ = 158.44
		$E_L^{(i)}$	13.2	12.0	11.0	10.1	9.2	8.4	7.7	7.0	6.5	5.9	
m=10 I=9 q=.44 p=.01		$E_R^{(i)}$	10.3	12.0	13.4	15.17	17.52	20	25	30	50	100	Ŧ = 363
		$E_L^{(i)}$	6.8	6.7	7.7	7.4	6.3	6.2	6.1	6.0	5.9	5.8	

TABLE 1: COMPARISON OF TIME TO RECEIVE i^{th} ROP, $E_R^{(i)}$, AND TO SEND i^{th} LABEL, $E_L^{(i)}$

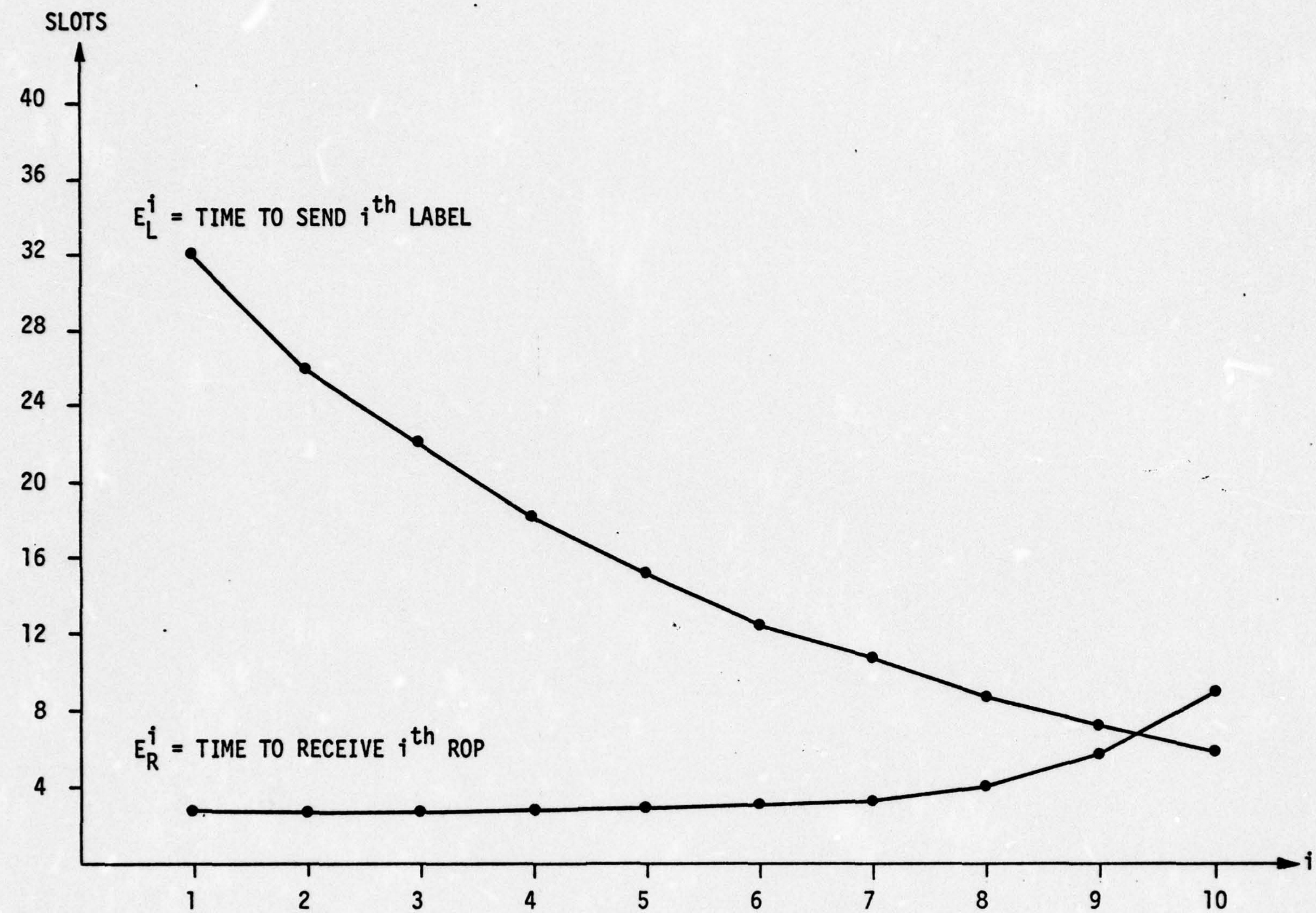


FIGURE 4: COMPARISON OF TIME TO RECEIVE ROP AND SEND LABEL 10 REPEATERS, 1 BUFFER, REPEATER RATE .10, REPEATER RATE .10, STATION RATE .44

9.3.4 Numerical Results For The Two Station Buffer Case

Figures 5, 6 and 7 depict the optimal parameter values and initialization time for complete interference, for I = 2, and no interference, respectively, for an m-repeater network. The optimal values were obtained by point-by-point search and are accurate to two significant digits.

Figure 8 shows the initialization time as a function of m, parameterized on I. The initialization time for a 1-buffer system is shown for comparison.

The following can be observed:

1. The optimal station transmission is nearly constant at $q^* \approx .40$, showing a slight tendency to decrease as m increases.

2. There is only a very small difference in the optimal station rate at low or high interference.

3. For complete interference $p^* < \frac{1}{m}$, and for large m, $p^* \approx \frac{.7}{m}$ (empirically determined).

4. As the interference decreases, p^* increases (as expected); again $p < \frac{1}{m}$ and $p^* \approx \frac{.8}{m}$ for large m.

5. As the interference decreases, the initialization time decreases. There is a reduction of about 15% in initialization time as we go from complete interference to zero interference.

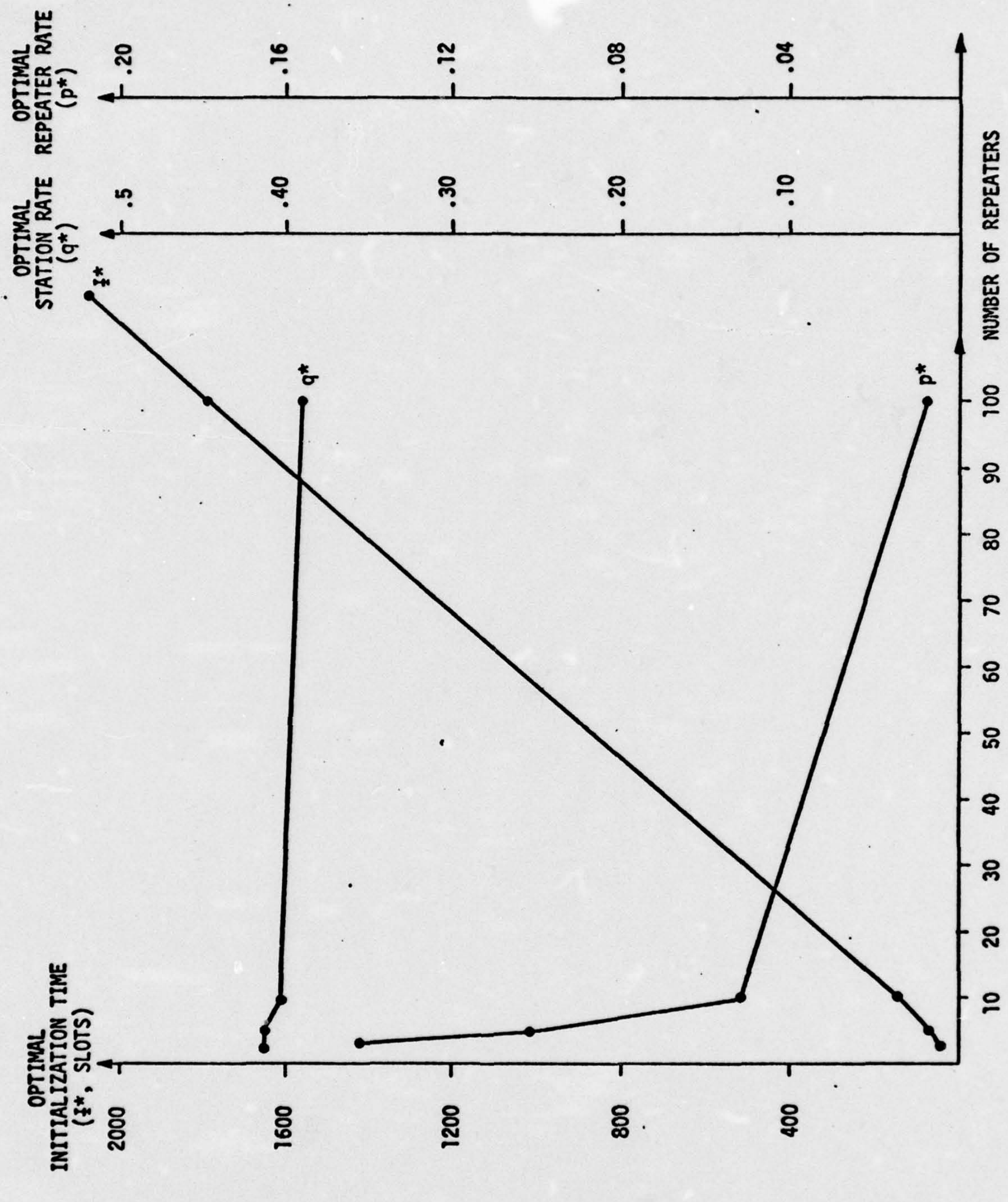


FIGURE 5: COMPLETE INTERFERENCE, TWO BUFFERS

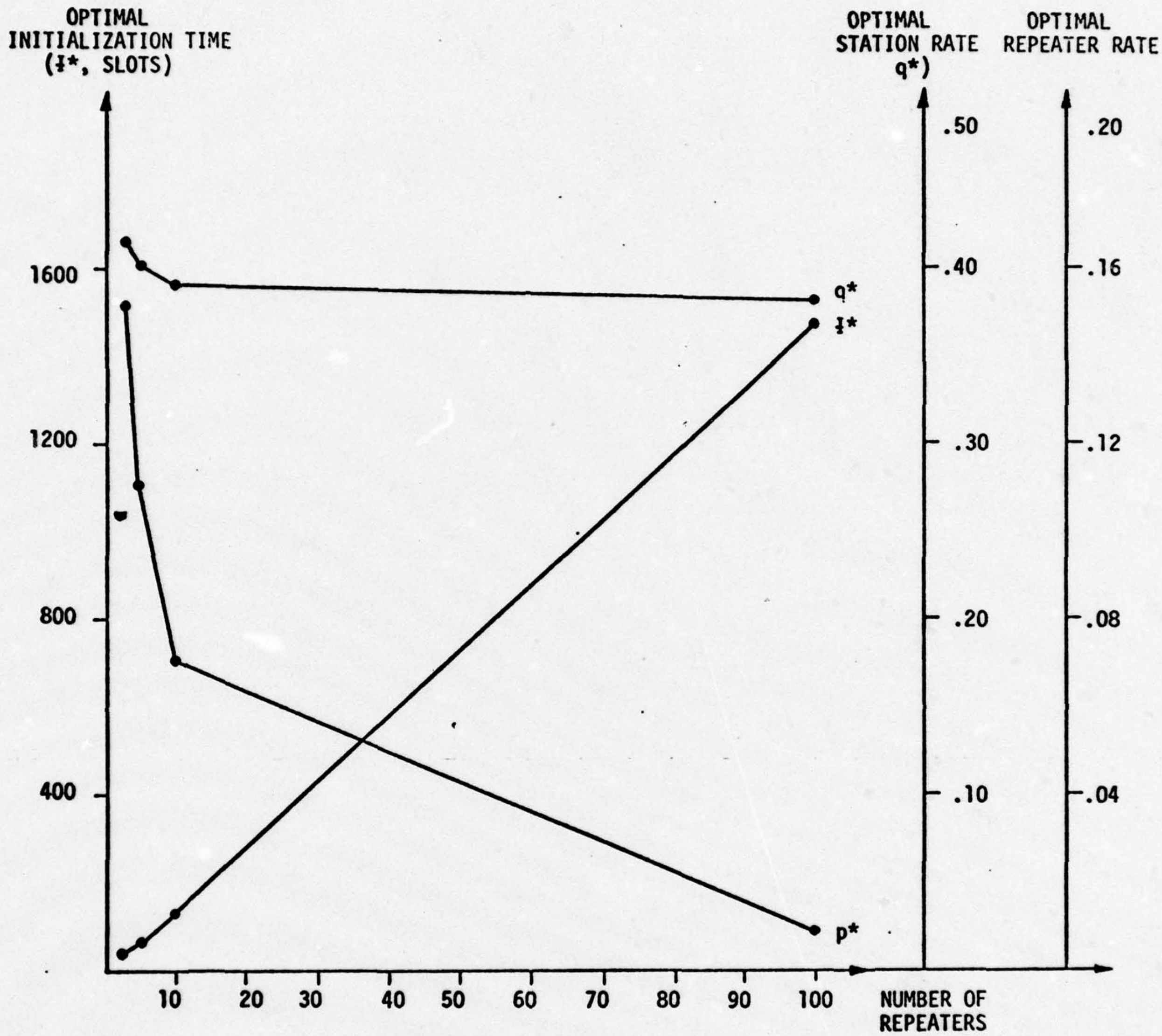


FIGURE 6: PARTIAL INTERFERENCE, I=2, 2 BUFFERS

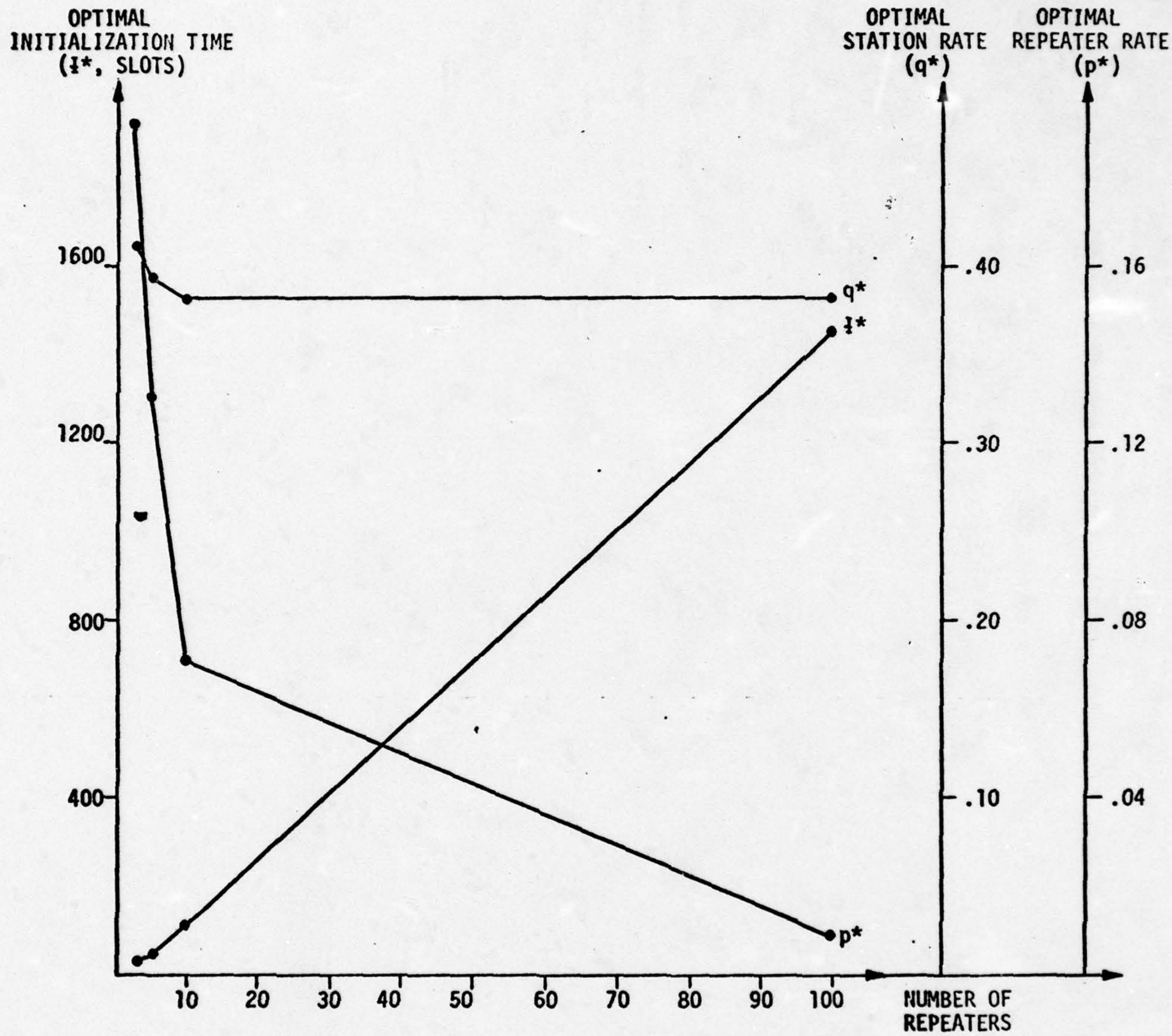


FIGURE 7: ZERO INTERFERENCE, 2 BUFFERS

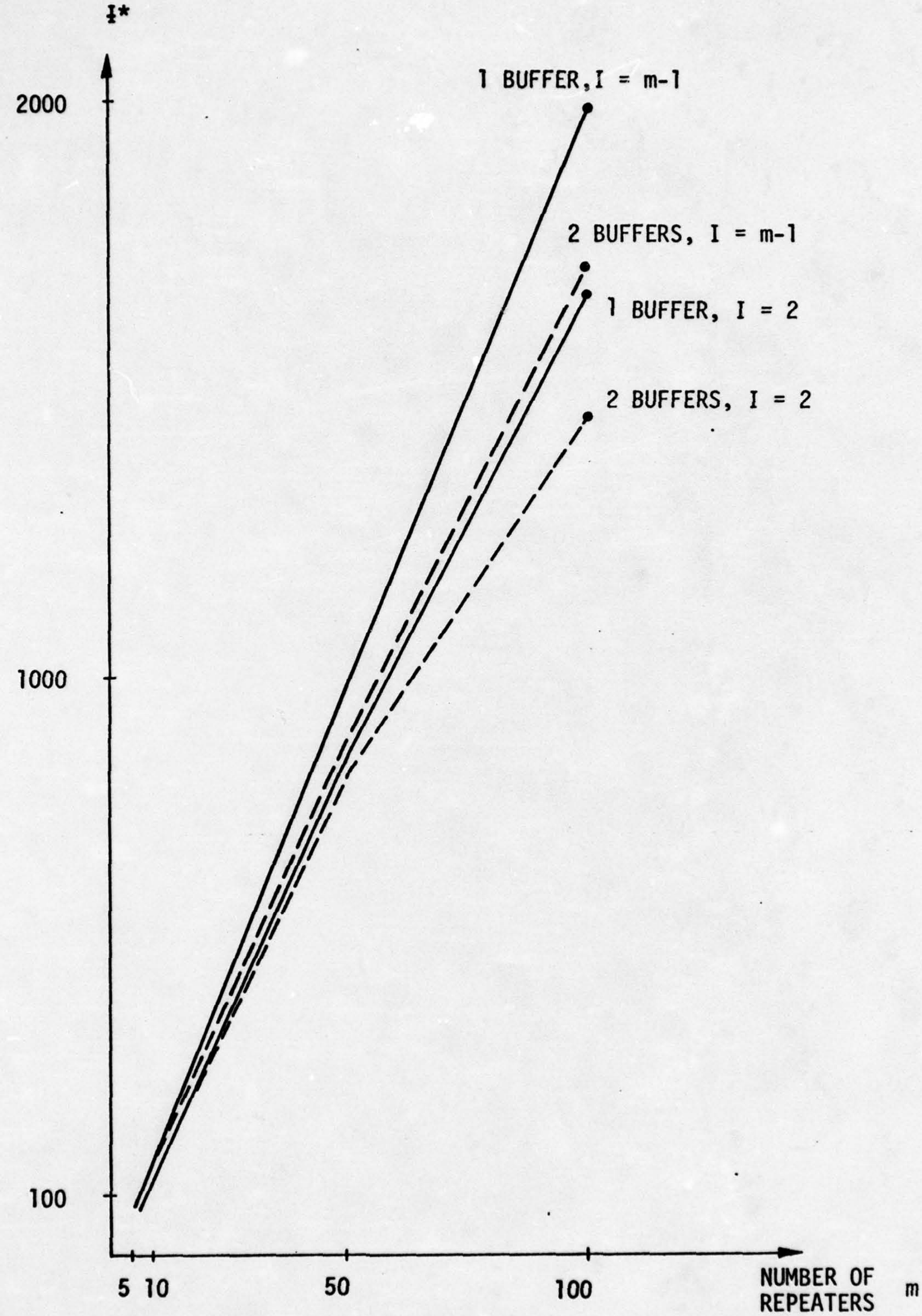


FIGURE 8: INITIALIZATION TIME AS A FUNCTION OF m, I AND b

9.3.5 Comparison of 1-Buffer and 2-Buffer Cases

A comparison between the above cases via Figures 1 to 8 shows:

1. The station's optimal transmission rate decreased 10% as we went from 1 buffer to 2 buffers. This is explained by the fact that in a 2-buffer situation there is no pressing urgency to clear the one occupied buffer since another is available for accepting ROP's.

2. The repeater's transmission rates are unchanged on the average, indicating that the repeaters can be ignorant of the station's buffer number.

3. The initialization time decreased as we went from 1 buffer to 2 buffers. Approximately 15% less time is required with two buffers than with one buffer.

4. Comparing these results with the results of Chapter 8, we observe that for a small number of repeaters, the degradation of changing from random selection of label transmission to FIFO is of the order of 1%. Such degradation, however, can be expected to increase as the network size increases.

9.4 COMPUTATIONAL COMPLEXITY

In Appendix A it is shown that the number of states in the Markov Chain for the b station buffer case is:

$$\sigma = \sum_{\ell=0}^{b} 3(m-\ell+1) = 3[mb+(b+1)(1-\frac{b}{2})] \quad (4)$$

Observations:

1. if $b = 1$, $\sigma = 3[m+1]$
2. if $b = 2$ $\sigma = 6[m]$
3. if $3<b<m$ $\sigma \leq 3\ bm$

Thus we see that in all cases $\sigma=O(bm)$, and for infinite buffer $(b=m)$, $\sigma=O(m^2)$. The improvement in the computational effort is shown in Table 2 which compares the number of states of this FIFO model for delivering labels to repeaters versus random service of the label queue (Chapter 8).

9.5 CONCLUSIONS

Closed form solution for the initialization time of a single-hop packet radio network were obtained. The solutions are for the case in which the station has one or two buffers for storing and sending labels and when it uses a first in first out queue management strategy. The slotted ALOHA access scheme was assumed. The computational complexity of the models was determined; the solutions were programed, and the optimum values of repeater and station transmission rates as a function of the interference pattern of repeaters were experimentally obtained. These optimum rates result in minimum initialization times.

The following conclusions emerge from the studies:

1. The initialization time with 2 buffers at the station is approximately 15% smaller than with 1 buffer, for the same interference pattern and network size.

2. The optimal station transmission rate, q*, is nearly independent of m(number of repeaters), for both values of the buffer size; however, q* decreased 10% as we went from 1 buffer to 2 buffers. Thus q is a function of the station's architecture only.

3. The optimal repeater transmission rates were independent of the buffer size on the average, indicating that the repeaters need not be aware of the station's buffer structure.

4. The optimal repeater transmission rate increases about 20% as interference goes from complete to zero; the initialization time decreases about 15% as we go from complete interference to zero interference.

5. In both cases for large m optimal repeater transmission rates were proportional to 1/m where m is number of repeaters in the network.

APPENDIX A: DERIVATION OF SOLUTION FOR THE ONE BUFFER CASE

A.1 Transition Probabilities

Because of the assumptions of complete interference and FIFO it is easy to compute the state transition matrix. For the complete interference case, a packet is successfully received by the station, if all other repeaters and the station are silent. Similarly, for a packet to be received successfully by a repeater it is required that all repeaters be silent.

We associate a status with each repeater during the initialization process. The state of the system is defined by the number of repeaters in each status. A repeater can have one of the following statuses:

1. Transmitting ROP's, but has not successfully sent one to the station.

2. Transmitting ROP's, but has already successfully sent one to the station.

3. Received label from station in previous slot, will forward ETE Ack to station in next slot.

4. Awaiting another label from station. It has received at least one label from the station but the subsequent ETE Ack('s) were unsuccessful. In this case the repeater is not transmitting ROP's since it has a label. However, the station will continue to transmit the label to the same repeater, since it did not receive an ETE Ack to the label.

5. Completed initialization - repeater has received label from station and forwarded a successful acknowledgement.

Given the present state of the chain, $(m_1, m_2, m_3, m_4, m_5)$, where m_i is the number of repeater in status i, the following list enumerates all possible transitions. We consider $m_3=0$ and $m_3=1$ separately.

1. No repeater received a label in the previous slot $(m_3=0)$

   a. Successful ROP when station queue is not empty. This requires $m_1>0$, $0<m_2+m_4<b$. Transition to state $(m_1-1, m_2+1, 0, m_4, m_5)$ occurs with probability

   $$(1-q)m_1\ p(1-p)^{m_1+m_2-1} = Z_1'.$$

   b. Successful ROP when station queue is empty. This requires $m_1>0$, $m_2+m_4=0$. Transition to state $(m_1-1, m_2+1, 0, m_4, m_5)$ with probability $m_1(p(1-p)^{m_1+m_2-1} = Z_1''$.

   Since Z_1' and Z_1'' represent probabilities of mutually exclusive events, we will use one notation Z_1 which will denote Z_1' or Z_1'', depending on the event.

   c. Successful label to a repeater with status 2. This cannot occur in the present model unless $m_4=0$. Transition to state $(m_1, m_2-1, 1, m_4, m_5)$ with probability $Z_2=(1-p)^{m_1+m_2}$.

d. Successful label to the repeater in status 4. This requires $m_4>0$. Transition to state $(m_1, m_2, 1, m_4-1, m_5)$ with probability $(1-p)^{m_1+m_2}q=Z_3$.

e. Else remain in same state. With probability $1-Z_1-Z_2-Z_3$.

2. Some repeater received a label in the previous slot. $(m_3=1,\ m_4=0)$

a. Successful ETE Ack. Transition to state $(m_1, m_2, 0, m_4, m_5+1)$ occurs with probability $(1-p)^{m_1+m_2}(1-q)$.

b. Unsuccessful ETE Ack; transition to state $(m_1, m_2, 0, m_4+1, m_5)$ with probability $1-(1-p)^{m_1+m_2}(1-q)$.

3. If the present state is $(0,0,0,0,m)$ go to state $(m,0,0,0,0)$ with probability 1.

It is elementary to verify that a unique stationary vector, $(\Pi_1, \Pi_2, \ldots \Pi_n)^T$, exists for this chain. Also, if the n^{th} state is completely labeled state, $(0,0,0,0,m)$, then the expected interarrival time between visits to state n is $1/\Pi_n$. Hence, the expected initialization time is $1/\Pi_n - 1$.

Alternatively, one could define T_i to be the time to reach the completely initialized state from state i. Suppose at the current

time the state of the Markov Chain is i and a transition to state j occurs (where j may equal i). Then it is easy to derive

$$E[T_i \mid \begin{smallmatrix}\text{next transition}\\ \text{is to } j\end{smallmatrix}] = 1 + E[T_j]. \tag{A.1}$$

Unconditioning we obtain

$$E[T_i] = \sum_j (1+E[T_j])\ P_{ij} = 1 + \sum_j E[T_j]\ P_{ij} \tag{A.2}$$

where P_{ij} is the i, j element of the state transition matrix. This results in a set of simultaneous equations that could be solved to obtain $E[T_i]$ for all i. In the following sections we obtain closed form solutions for these equations with 1 and 2 buffers.

A.2 Number of States in the Chain

Consider the quintuple $(m_1, m_2, m_3, m_4, m_5)$. Let ℓ ROP's be in the buffer (i.e., $\ell = m_2+m_3+m_4$) and m-ℓ be in state 1 or 5. Since

1. $m_3 \leq 1$
2. $m_4 \leq 1$
3. $m_3 + m_4 \leq 1$

We see that given ℓ elements, there are only three ways in which to partition them according to statuses 2, 3 and 4, namely,

1.	$m_2 = \ell-1$	$m_3 = 0$	$m_4 = 1$
2.	$m_2 = \ell-1$	$m_3 = 1$	$m_4 = 0$
3.	$m_2 = \ell$	$m_3 = 0$	$m_4 = 0$

On the other hand there are $m-\ell+1$ ways the $m-\ell$ elements can be partitioned in the two statuses, 1 and 5. Thus, if σ represents the number of possible states of the above chain, by summing over all allowable values of ℓ, we find

$$\begin{aligned} \sigma &= \sum_{\ell=0}^{b} 3(m-\ell+1) = 3[mb - \frac{b(b+1)}{2} + (b+1)] \\ &= 3[mb + (b+1)(1 - \frac{b}{2})] \end{aligned} \tag{A.3}$$

A.3 Solution to 1-Buffer Markov Chain Model

By a cycle of a Markov Chain we mean a section of the state transition diagram. We consider cycles C_i such that

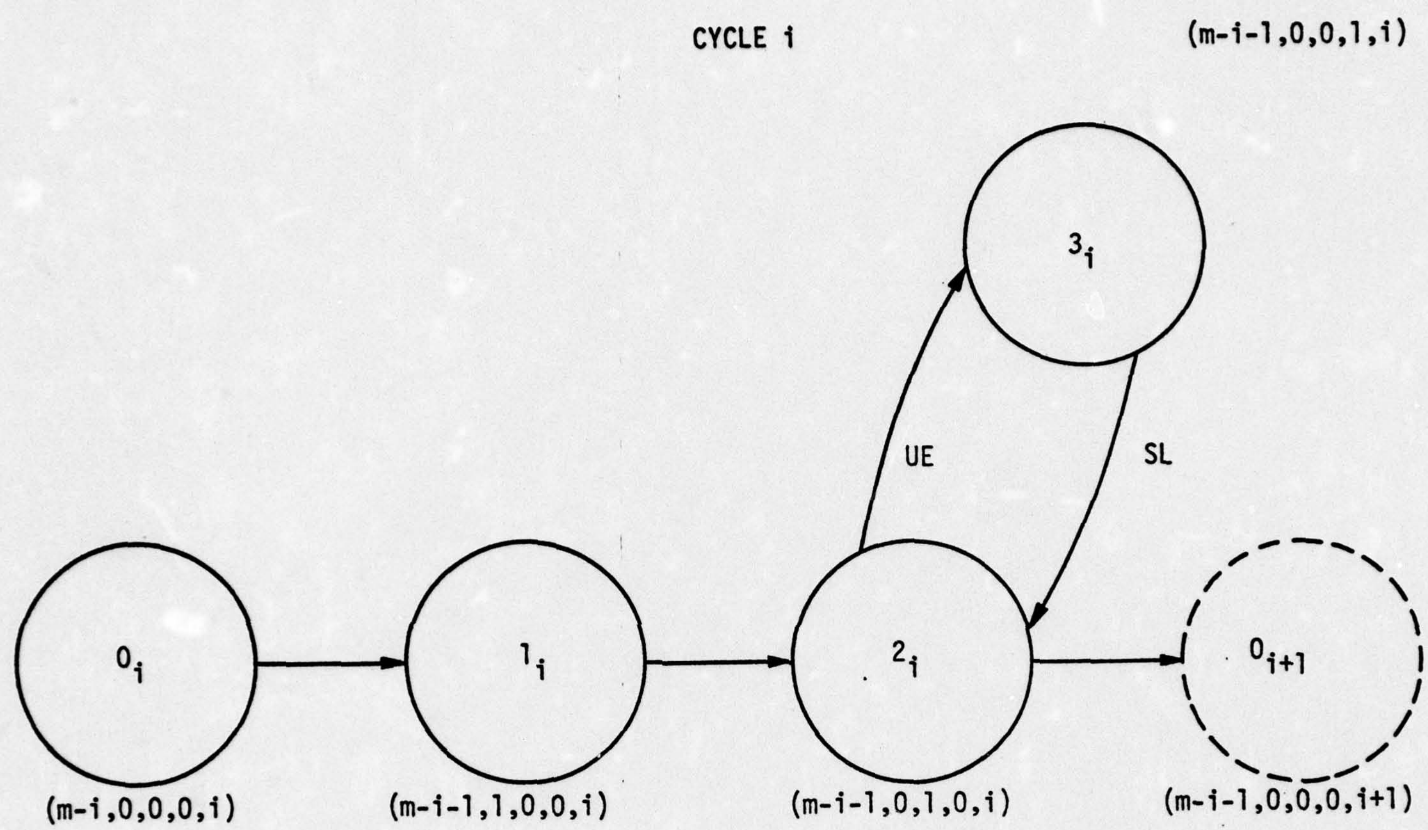
1. The C_i's are disjoint.

2. Any state of the chain belongs to one cycle.

3. C_i and C_{i+1}, when considered as graphs, are isomorphic for all but possibly the first and last case.

The solution strategy involves computing the expected time to traverse one cycle, and then summing these over cycles 0,1,...,m-1. To clarify the analytical procedure to be followed we illustrate in Figure 9 a cycle of the state transition diagram for the 1-buffer case. SR, SL, SE, UE represent successful ROP, successful label, successful ETE Ack and unsuccessful ETE Ack, respectively.

The cycle starts with the state (m-i, 0,0,0,i) representing m-i unlabeled repeaters and i labeled repeaters. The following states are depicted in Figure 9:



SR = Successful ROP; SL = Successful Label; SE = Successful ETE Ack

UE = Unsuccessful ETE Ack

FIGURE 9: CYCLE i FOR 1-BUFFER CASE

- 0_i = chain at beginning of cycle i = (m-i, 0,0,0,i)
- 1_i = chain after SR = (m-i-1,1,0,0,i)
- 2_i = chain after SL = (m-i-1,0,1,0,i)
- 3_i = chain after UE = (m-i-1,0,0,1,i)
- 0_{i+1} = chain at next cycle = chain after SE = (m-i-1,0,0,0,i+1)

The simplification that allows us to solve the 1 buffer and 2 buffer models with relative ease is that in each case there are m+1 states that must be entered in moving from state (m,0,0,0,0) to state (0,0,0,0,m). By defining cycles in the appropriate, we take advantage of this. For buffer sizes greater than 2, this is not true.

Note that for the 1 buffer case, the expected time to initialize the last m-k repeaters (after k repeaters have been initialized) equals the expected time to initialize a network with k repeaters. This is not true for larger buffer sizes.

Let $E[\mathcal{I}_i]$ be the expected time to go through cycle i; then if $\mathcal{I}$ represents the expected initialization time,

$$\mathcal{I} = \sum_{i=0}^{m-1} E[\mathcal{I}_i] \tag{A.4}$$

Throughout the analysis we use the fact that the geometrical distribution is memoryless; that is, if τ is a geometrically distributed random variable; then

$$\text{Prob}(\tau > i+j \mid \tau > i) = \text{Prob}(\tau > j) \tag{A.5}$$

for i,j integers.

In our case τ is the time spent in a specific state of the Markov Chain; the time to leave any state is geometrically distributed.

Consider the following particular scenario. The chain is in state A. If event E occurs transition to state B occurs; else if $\overline{E}$ (its complement) occurs go to state C. Remain in state C until some other event F occurs then return to state A. See Figure 10. Let $E[T_{AB}]$ and $E[T_{CA}]$ be the expected time to go from A to B and C to A, respectively. Because of the abovementioned memoryless property:

$$E[T_{AB} \mid \text{outcome of event}] = \begin{cases} 1 & , \text{ if } E \\ 1 + E[T_{CA}] + E[T_{AB}] & , \text{ if } \overline{E} \end{cases} \qquad (A.6)$$

unconditioning, if p(E) = prob of event E,

$$E[T_{AB}] = p(E) + (1-p(E)) \; \{1 + E[T_{CA}] + E[T_{AB}]\} \qquad (A.7)$$

from which one obtains:

$$E[T_{AB}] = \frac{1+(1-p(E))E[T_{CA}]}{p(E)} \qquad (A.8)$$

Going back to our Markov Chain, let $E[T_{x_i y_i}]$ be the expected time to go from state x to state y in cycle i.

Then clearly from Figure 9:

$$E(\bar{I}_i) = E[T_{0_i 1_i}] + E[T_{1_i 2_i}] + E[T_{2_i 0_{i+1}}] \qquad (A.9)$$

We examine each component separately. In this analysis let $P_{x_i y_i}$ be the transition probability of going from state x in cycle i to state y in cycle i.

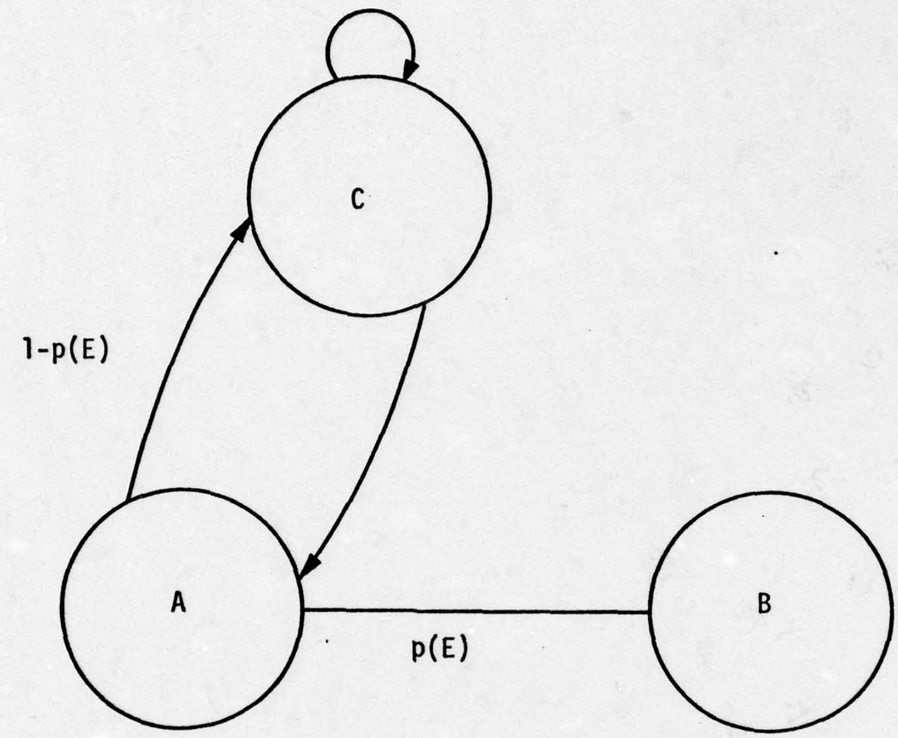


FIGURE 10: TRANSITION TIME UNDER MEMORYLESS DISTRIBUTIONS

A.3.1 Evaluation of $E[T_{0_i 1_i}]$

A transition from state 0_i to state 1_i represents a successful ROP reception; this can occur only if

1. The station is silent.

2. m-i-1 repeaters are silent, and one is broadcasting.

Thus the probability of this transition is:

$$P_{0_i 1_i} = (m-i)\, p(1-p)^{m-i-1} \quad , \; i=0,1,\ldots,m-1 \qquad \text{(A.10)}$$

Finally since the distribution of time spent in state 0_i is geometric we obtain:

$$E[T_{0_i 1_i}] = \frac{1}{P_{0_i 1_i}} = \frac{1}{(m-i)p(1-p)^{m-i-1}} \qquad \text{(A.11)}$$

A.3.2 Evaluation of $E[T_{1_i 2_i}]$

A transition from state 1_i to state 2_i represents a successful label delivery; this can occur only if

1. The station transmits.

2. The repeater in question is silent.

3. All the repeaters which can interfere with the repeater in question are silent.

Thus the probability of this transition is:

$$P_{1_i 2_i} = q(1-p)^{I(i)+1} \tag{A.12}$$

where I(i) is the number of interfering repeaters at cycle i. For complete interference I(i) = m-i for all repeaters and our result is exact. For non-complete interference $I(i) \leq \min(I,m-i)$. Thus for the latter case one is restricted to an approximation. Two conceivable approximations are:

$$I(i) \approx \min(I,m-i) \tag{A.13}$$

$$I(i) + 1 \approx (I+1)\frac{m-i+1}{m} \tag{A.14}$$

The second approximation has been implemented into our computational routines and is subsequently referred to as the linear decrease approximation.

Now, since the distribution of $T_{1_i 2_i}$ is geometric,

$$E[T_{1_i 2_i}] = \frac{1}{q(1-p)^{I(i+1)}} \tag{A.15}$$

A.3.3 Evaluation of $E[T_{2_i 0_{i+1}}]$

This is the only case that needs some thought. Note that from state 2_i we can go directly to state 0_{i+1}, if the ETE Ack transmission is successful, and to state 3_i if the ETE Ack transmission is unsuccessful. We condition on this event. In this case we can apply the analysis developed for the scenario depicted in Figure 10; doing so, we obtain:

$$E[T_{2_i 0_{i+1}}] = \frac{1+(1-P_{2_i 0_{i+1}})E[T_{3_i 2_i}]}{P_{2_i 0_{i+1}}}. \tag{A.16}$$

Since such an ETE Ack is received if all active repeaters and the station are silent, we have:

$$P_{2_i 0_{i+1}} = (1-p)^{m-i-1}(1-q) \tag{A.17}$$

Note that

$$P_{3_i 2_i} = q(1-p)^{I(i)} \tag{A.18}$$

since all interfering repeaters must be silent and the station must broadcast. Thus the expected time is:

$$E[T_{3_i 2_i}] = \frac{1}{q(1-p)^{I(i)}} \tag{A.19}$$

because the distribution of the time spent in state 3_i is geometric. We thus have

$$E[T_{2_i 0_{i+1}}] = \frac{1}{(1-p)^{m-i-1}(1-q)} \{1 + \frac{[1-(1-p)^{m-i-1}(1-q)]}{q(1-p)^{I(i)}}.\} \tag{A.20}$$

Finally, the total expected initialization time is obtained by summing the three quantities obtained above, over every cycle.

$$\bar{T} = \sum_{i=0}^{m-1} \{\frac{1}{(m-i)p(1-p)^{m-i-1}} + \frac{1}{q(1-p)^{I(i)+1}} + \frac{1}{(1-q)(1-p)^{m-i-1}} (1+\frac{[1-(1-q)(1-p)^{m-i-1}]}{q(1-p)^{I(i)}})\} \tag{A.21}$$

We have thus derived the exact solution for the 1-buffer Markovian model with complete interference and an approximate solution for the general interference case.

Conceptually the same approach can be applied to treat the general interference case, but the development is very laborious. In this case it is necessary to know specifically:

(1) Which repeaters have already been initialized prior to the i^{th} cycle.

(2) Which repeater is labeled during the i^{th} cycle given item (1).

(3) Expected time to complete the i^{th} labeling, given (2).

Then in order to compute total expected initialization time, it is necessary to compute probabilities for items (1) and (2) for all possible combinations. A cursory analysis indicates that computing probabilities for item (2) and expected time in (3) is relatively easy, but computing the probabilities for (1) would be laborious. Most likely this would have to be computerized (rather than deriving a closed form solution), and even then, because of the computational complexity, only small networks could be analyzed.

APPENDIX B: DERIVATION OF SOLUTION FOR A 2 BUFFER MODEL WITH FIFO SERVICE DISCIPLINE

B.1 General Strategy

It has been stated above that close form computations are relatively easy when the number of states by which one can enter a cycle is one. To achieve this, we must alter slightly the definition of cycle employed in Appendix A. Let

Cycle -1	start with	(m,0,0,0,0)
	end with	(m-1,1,0,0,0)
Cycle i	start with	(m-i-1,1,0,0,i)
	end with	(m-i-2,1,0,0,i+1), i=0,1,2,...,m-2
Cycle m-1	start with	(0,1,0,0,m-1)
	end with	(0,0,0,0,m)

Note that Cycle -1 involves only the successful sending of a ROP to the station while the cycle m-1 involves only the labeling of the last repeater after all others have been labeled, after this repeater has successfully sent a ROP.

Thus at the beginning of Cycle i, i=0,1,...,m-2, there are m-i-1 repeaters with status 1, 1 repeater with status 2, and i repeaters with status 5.

Figure 11 illustrates the states for a typical cycle $0 \le i \le m-2$. The same nomenclature of the previous section has been employed in Figure 11; notice that now there are three separate classes of routes which the chain can take to traverse the cycle. Each cycle starts with 1 ROP in the buffer. Either one of the following classes of routes can be taken:

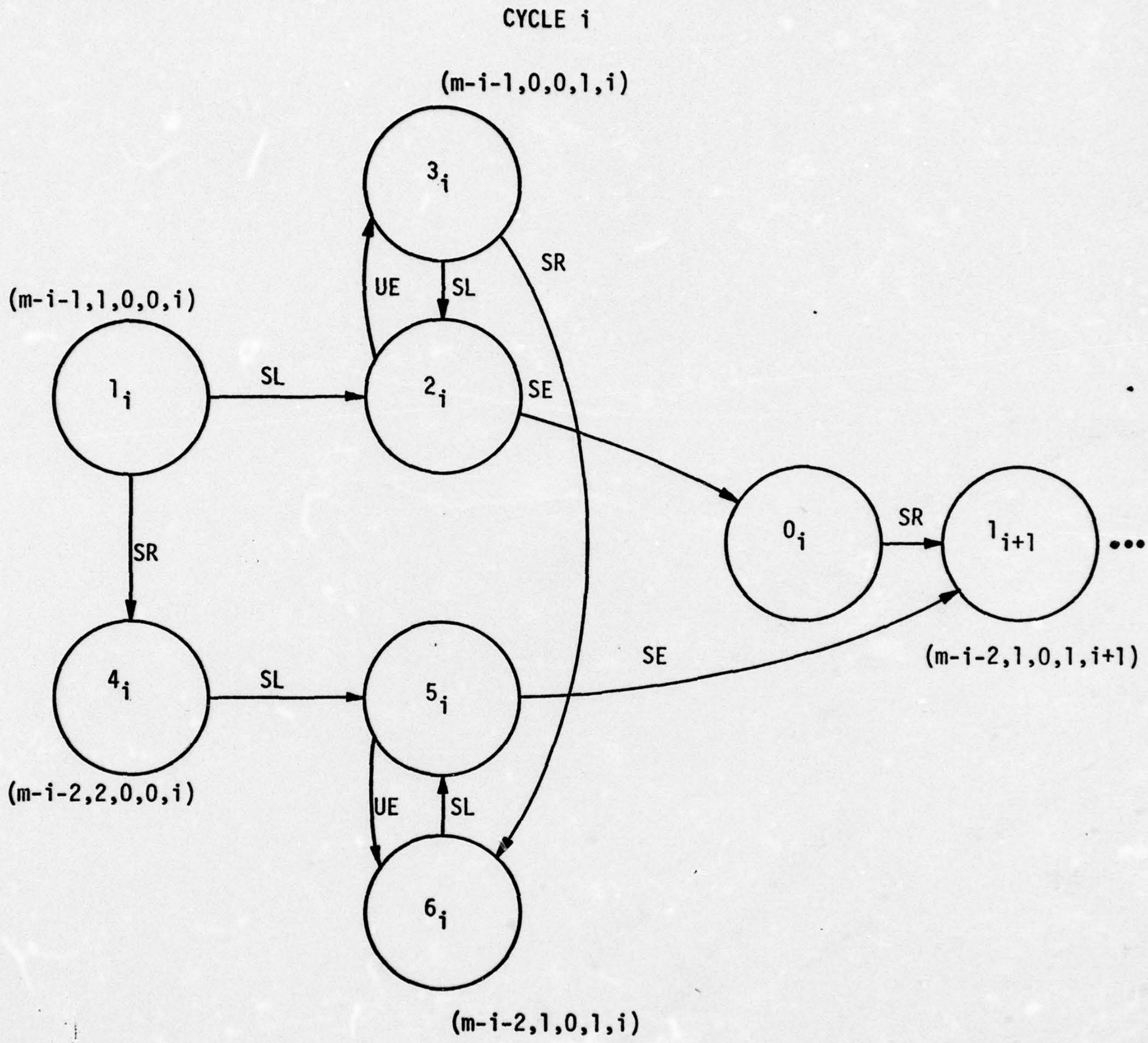


FIGURE 11: CYCLE i, i=0,1,...m-2

1. Another ROP is received by the station before a label is recieved by a repeater (follow the lower route passing through states 4_i, 5_i, and possibly 6_i).

2. The label is received and acknowledged by the repeater before another ROP is received (upper route without cross over passing through states 2_i, 0_i and possibly 3_i).

3. The label is dispatched several times but before it is successfully acknowledged another ROP is received (upper route with cross over passing through states 2_i, 3_i, 6_i and 5_i).

Our calculations involve examining the expected time to traverse each of these classes of routes, and deciding with what probability each is chosen.

As before the total expected initialization time is the sum of the expected times required to traverse each cycle:

$$\mathcal{T} = \sum_{i=-1}^{m-1} E[\mathcal{T}_i] \qquad \text{(B.1)}$$

Since cycle -1, and m-1 are different, we address these first.

B.2 Cycles -1 and m-1

Figure 12 depicts that section of the state transition diagram for cycle -1 and m-1.

B.2.1 Evaluation of $E[\mathcal{T}_{-1}]$

This involves a SR, given that all m repeaters are uninitialized; this requires one repeater to transmit and all others to remain silent. Thus the expected time of cycle -1 is:

$$E[\mathcal{T}_{-1}] = \frac{1}{P_{0_{-1}1_0}} = \frac{1}{mp(1-p)^{m-1}} \tag{B.2}$$

B.2.2 Evaluation of $E[\mathcal{T}_{m-1}]$

Observe that, except for the number of unlabeled repeaters, this section of the state diagram is the same as a cycle for a 1-buffer problem. As in the previous section

$$\begin{aligned} E[\mathcal{T}_{m-1}] &= E[T_{1_{m-1}2_{m-1}}] + E[T_{2_{m-1}0_{m-1}}] \\ &= \frac{1}{q(1-p)} + \frac{1}{P_{2_{m-1}0_{m-1}}} \{1 + E[T_{3_{m-1}2_{m-1}}]\ (1-P_{2_{m-1}0_{m-1}})\} \end{aligned} \tag{B.3}$$

where $P_{2_{m-1}0_{m-1}}$ = prob (ETE Ack is successful.)

Since every repeater is silent at this stage, we only require that the station be silent; thus $P_{2_{m-1}0_{m-1}} = (1-q)$, giving the expected time for cycle m-1:

$$E[\mathcal{T}_{m-1}] = \frac{1}{q(1-p)} + \frac{2}{1-q} \tag{B.4}$$

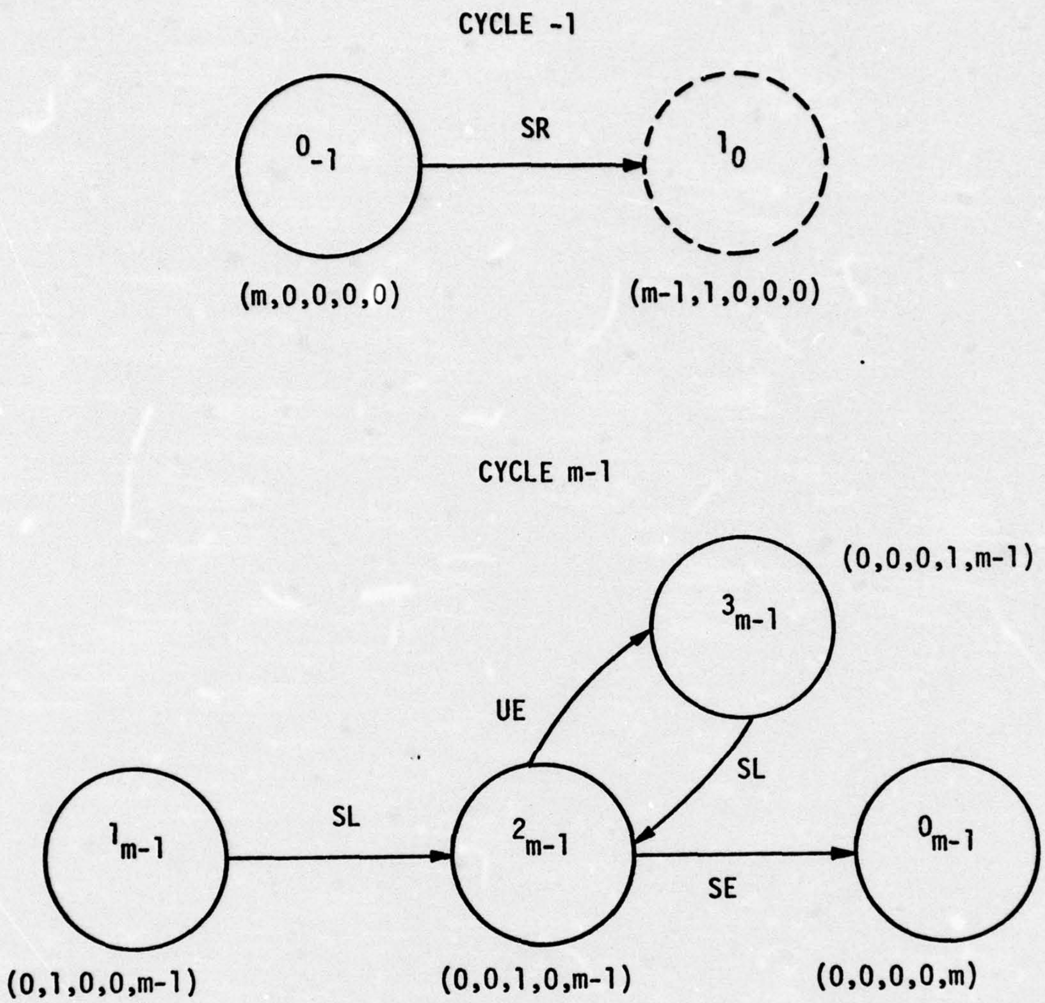


FIGURE 12: CYCLE -1 AND m-1

B.3 Evaluation of $E[\bar{T}_i]$, $i=0,1,\ldots,m-2$

For a typical cycle i, we are interested in quantifying the time needed to traverse it. Such time is made up of various components, as indicated below.

B.3.1 Computation of $E[T_{1_i 1_{i+1}}]$

To compute this expression assume that at the current slot the state of the chain is 1_i, then condition on the outcome of the next slot. Reasoning as in Appendix A, it is easy to derive

$$P_{1_i 2_i} = \text{Prob (go from } 1_i \text{ to } 2_i) = q(1-p)^{1+I(i)} \tag{B.5}$$

$$P_{1_i 4_i} = \text{Prob (go from } 1_i \text{ to } 4_i) = (1-q)(1-p)^{m-i-1} p(m-i-1) \tag{B.6}$$

Then, referring to Figure 11, we see that

$$E\left[T_{1_i 1_{i+1}} \,\middle|\, \begin{array}{l}\text{outcome of}\\ \text{next slot}\end{array}\right] = \begin{cases} 1 + E[T_{2_i 1_{i+1}}] & \text{if a label is delivered} \\ 1 + E[T_{4_i 1_{i+1}}] & \text{if a ROP is received} \\ 1 + E[T_{1_i 1_{i+1}}] & \text{if neither of the above} \end{cases} \tag{B.7}$$

where we have made use of the memoryless property described above.

Unconditioning we obtain:

$$E[T_{1_i 1_{i+1}}] = (1 + E[T_{2_i 1_{i+1}}])\, P_{1_i 2_i} + (1 + E[T_{4_i 1_{i+1}}])\, P_{1_i 4_i}$$

$$+ (1 + E[T_{1_i 1_{i+1}}])(1-P_{1_i 2_i} - P_{1_i 4_i}) \tag{B.8}$$

Solving for $E[T_{1_i 1_{i+1}}]$, we obtain:

$$E[T_{1_i 1_{i+1}}] = \frac{1 + P_{1_i 2_i}\, E[T_{2_i 1_{i+1}}] + P_{1_i 4_i}\, [T_{4_i 1_{i+1}}]}{P_{1_i 2_i} + P_{1_i 4_i}} \tag{B.9}$$

We notice that to proceed we need $E[T_{2_i 1_i +1}]$ and $E[T_{4_i 1_{i+1}}]$; we address $E[T_{4_i 1_{i+1}}]$ next.

B.3.2 Computation of $E[T_{4_i 1_{i+1}}]$

Referring to Figure 11, we see that

$$E[T_{4_i 1_{i+1}}] = E[T_{4_i 5_i}] + E[T_{5_i 1_{i+1}}] \tag{B.10}$$

We now evaluate the epxressions on the right-hand side of Eq. (B.10). It is quite simple to show that:

$$E[T_{4_i 5_i}] = \frac{1}{P_{4_i 5_i}} = \frac{1}{q(1-p)^{1+I(i)}} \tag{B.11}$$

since we require a successful label delivery from the state (m-i-2,2,0,0,i).

To obtain $E[T_{5_i 1_{i+1}}]$ we must condition on the outcome of the next slot, namely whether the ETE Ack is successful or not. Using the analysis applied to Figure 9 in Appendix A we find:

$$E[T_{5_i 1_{i+1}}] = \frac{1}{P_{5_i 1_{i+1}}} \{1 + E[T_{6_i 5_i}](1-P_{5_i 1_{i+1}})\} \tag{B.12}$$

Note that:

$$P_{5_i 1_{i+1}} = (1-p)^{m-i-1}(1-q) \tag{B.13}$$

since we are in state (m-i-2,1,1,0,i), and the station and all other unlabeled repeaters must be silent for the ETE to be successful. Reasoning as in Appendix A Section 3, we obtain:

$$E[T_{6_i 5_i}] = \frac{1}{P_{6_i 5_i}} = \frac{1}{q(1-p)^{I(i)}} \tag{B.14}$$

since one less repeater is now active after state 6_i is reached.

Finally:

$$E[T_{4_i 1_{i+1}}] = \frac{1}{q(1-p)^{1+I(i)}} + \frac{1}{(1-p)^{m-i-1}(1-q)}$$

$$\{1 + [1-(1-p)^{m-i-1}(1-q)] \frac{1}{q(1-p)^{I(i)}}\} \tag{B.15}$$

B.3.3 Evaluation of $E[T_{2_i 1_{i+1}}]$

Assuming the current state of the chain to be 2_i and conditioning on the event of the next slot, namely whether the ETE Ack is successful, we immediately see

$$E[T_{2_i 1_{i+1}} \mid \text{event of next slot}] = \begin{cases} 1 + E[T_{0_i 1_{i+1}}] & \text{if ETE Ack successful} \\ 1 + E[T_{3_i 1_{i+1}}] & \text{if ETE Ack unsuccessful} \end{cases} \tag{B.16}$$

Note that (see Figure 11):

$$P_{2_i 0_i} = \text{prob (ETE Ack is successful from state } 2_i) =$$

$$= (1-q)(1-p)^{m-i-1} \tag{B.17}$$

Unconditioning, we get:

$$E[T_{2_i 1_{i+1}}] = P_{2_i 0_i}(1 + E[T_{0_i 1_{i+1}}])$$

$$+ (1 + E[T_{3_i 1_{i+1}}])(1-p_{2_i 0_i}) \tag{B.18}$$

Since

$$E[T_{0_i 1_{i+1}}] = \frac{1}{(m-i-1)p(1-p)^{m-i-2}} \tag{B.19}$$

(due to the fact that the station is automatically silent when the label queue is empty) we obtain:

$$E[T_{2_i 1_{i+1}}] = 1 + P_{2_i 0_i} \frac{1}{(m-i-1)p(1-p)^{m-i-2}}$$

$$+ (1-P_{2_i 0_i})\, E[T_{3_i 1_{i+1}}] \tag{B.20}$$

Thus we need to find $E[T_{3_i 1_{i+1}}]$.

B.3.4 Evaluation of $E[T_{3_i 1_{i+1}}]$

Note this calculation is more complicated than the single buffer case because it is possible to transfer from 3_i to 6_i, i.e., another ROP is received before label delivery. Again we condition on the next slot.

$$E[T_{3_i 1_{i+1}} \mid \text{event of next slot}] = \begin{cases} 1 + E[T_{2_i 1_{i+1}}] & \text{, if label successful} \\ 1 + E[T_{6_i 1_{i+1}}] & \text{, if ROP successful} \\ 1 + E[T_{3_i 1_{i+1}}] & \text{, if neither} \end{cases} \tag{B.21}$$

Unconditioning:

$$E[T_{3_i 1_{i+1}}] = (1 + E[T_{2_i 1_{i+1}}])\, P_{3_i 2_i} + (1 + E[T_{6_i 1_{i+1}}])\, P_{3_i 6_i} + (1 + E[T_{3_i 1_{i+1}}])(1 - P_{3_i 1_{i+1}} - P_{3_i 6_i}) \tag{B.22}$$

where

$$P_{3_i 6_i} = \text{Prob (ROP is successful from state } 3_i)$$
$$= (m-i-1)p(1-p)^{m-i-2}(1-q) \tag{B.23}$$

$$P_{3_i 2_i} = \text{Prob (label is successful from state } 3_i)$$
$$= q(1-p)^{I(i)} \tag{B.24}$$

Solving, we obtain:

$$E[T_{3_i 1_{i+1}}] = \frac{1 + P_{3_i 2_i} E[T_{2_i 1_{i+1}}] + P_{3_i 6_i} E[T_{6_i 1_{i+1}}]}{P_{3_i 2_i} + P_{3_i 6_i}} \qquad \text{(B.25)}$$

$E[T_{6_i 1_{i+1}}]$ can be written as:

$$E[T_{6_i 1_{i+1}}] = E[T_{6_i 5_i}] + E[T_{5_i 1_{i+1}}] \qquad \text{(B.26)}$$

where:

$$E[T_{6_i 5_i}] = \frac{1}{q(1-p)^{I(i)}} \qquad \text{(B.27)}$$

and $E[T_{5_i 1_{i+1}}]$ was derived in B.3.2.

Hence we obtained an equation for $E[T_{3_i 1_{i+1}}]$ in terms of $E[T_{2_i 1_{i+1}}]$. Together with the equation of subsection 3, we have a system which must be solved simultaneously.

B.3.5 Evaluation of $E[T_i]$: The Final Result

We now pull things together; from subsection B.3.1 we have:

$$E[T_{1_i 1_{i+1}}] = \frac{1}{P_{1_i 2_i} + P_{1_i 4_i}} + \frac{P_{1_i 2_i}}{P_{1_i 2_i} + P_{1_i 4_i}} E[T_{2_i 1_{i+1}}] + \frac{P_{1_i 4_i}}{P_{1_i 2_i} + P_{1_i 4_i}} E[T_{4_i 1_{i+1}}] \qquad \text{(B.28)}$$

From subsection B.3.2 we have:

$$E[T_{4_i 1_{i+1}}] = E[T_{4_i 5_i}] + \frac{1}{P_{5_i 1_{i+1}}} \{1 + E[T_{6_i 5_i}] (1-P_{5_i 1_{i+1}})\} \quad \text{(B.29)}$$

We only need $E[T_{2_i 1_{i+1}}]$. From subsections B.3.3 and B.3.4 we have:

$$E[T_{2_i 1_{i+1}}] = 1 + P_{2_i 0_i} E[T_{0_i 1_{i+1}}] + (1-P_{2_i 0_i}) \cdot E[T_{3_i 1_{i+1}}] \quad \text{(B.30)}$$

and

$$E[T_{3_i 1_{i+1}}] = \frac{1}{P_{3_i 6_i} + P_{3_i 2_i}} + \frac{P_{3_i 2_i}}{P_{3_i 6_i} + P_{3_i 2_i}} E[T_{2_i 1_{i+1}}] + \frac{P_{3_i 6_i}}{P_{3_i 6_i} + P_{3_i 2_i}} E[T_{6_i 5_i}] + E[T_{5_i 1_{i+1}}] \quad \text{(B.31)}$$

The following procedure is employed:

1. Substitute $E[T_{5_i 1_{i+1}}]$ (which has been computed above) into equation (B.31).

2. Solve the system of equations (B.30) and (B.31) for $E[T_{2_i 1_{i-1}}]$.

3. The answer to 2 is in terms of $E[T_{6_i 5_i}]$ which was derived above; substitute such expression.

We thus have $E[T_{2_i 1_{i+1}}]$, namely:

$$E[T_{2_i 1_{i+1}}] =$$

$$= \{1 + (1-q)(1-p)^{m-i-1} \frac{1}{(m-i-1)p(1-p)^{m-i-2}} + [1-(1-q)(1-p)^{m-i-1}].$$

$$[\frac{1}{(m-i-1)p(1-p)^{m-i-2}(1-q) + q(1-p)^{I(i)}} + \frac{1}{(1-p)^{m-i-1}(1-q)} \cdot$$

$$\frac{(m-i-1)p(1-p)^{m-i-2}(1-q)}{(m-i-1)p(1-p)^{m-i-2}(1-q) + q(1-p)^{I(i)}} [1 + \frac{1}{q(1-p)^{I(i)}}]]\}$$

$$\{1 - \frac{[1-(1-q)(1-p)^{m-i-1}.q(1-p)^{I(i)}}{(m-i-1)p(1-p)^{m-i-2}(1-q)+q(1-p)^{I(i)}}\}^{-1} \qquad \text{(B.32)}$$

Substitute Equations (B.29) and (B.32) into Equation (B.28) we obtain:

$$E[T_{1_i 1_{i+1}}]^* = \frac{1}{p(m-i-1)(1-q)(1-p)^{m-i-1} + q(1-p)^{1+I(i)}} +$$

$$\frac{q(1-p)^{1+I(i)}}{p(m-i-1)(1-q)(1-p)^{m-i-1} + q(1-p)^{1+I(i)}}$$

$$\{1 + \frac{(1-q)(1-p)^{m-i-1}}{(m-i-1)p(1-p)^{m-i-2}} + [1-(1-q)(1-p)^{m-i-1}].$$

$$[\frac{1}{(m-i-1)p(1-p)^{m-i-2}(1-q) + q(1-p)^{I(i)}}$$

*Equation continues on next page.

$$+ \frac{1}{(1-p)^{m-i-1}(1-q)} \cdot$$

$$\frac{(m-i-1)p(1-p)^{m-i-2}(1-q)}{(m-i-1)p(1-p)^{m-i-2}(1-q) + q(1-p)^{I(i)}} [1 + \frac{1}{q(1-p)^{I(i)}}]]\}$$

$$\{1 - \frac{[1-(1-q)(1-p)^{m-i-1}]\cdot q(1-p)^{I(i)}}{(m-i-1)p(1-p)^{m-i-2}(1-q) + q(1-p)^{I(i)}}\}^{-1}$$

$$+ \frac{(m-i-1)p(1-q)(1-p)^{m-i-1}}{p(m-i-1)(1-q)(1-p)^{m-i-1} + q(1-p)^{1+I(i)}} \cdot$$

$$\{\frac{1}{q(1-p)^{1+I(i)}} + \frac{1}{(1-p)^{m-i-1}(1-q)} \cdot$$

$$[1+(1-p)^{m-i-1}(1-q)] \frac{1}{q(1-p)^{I(i)}}]\} \qquad \text{(B.33)}$$

Thus we have accomplished our goal of determining the expected time to traverse a typical cycle. Summing over all cycles gives the total expected initialization time.

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